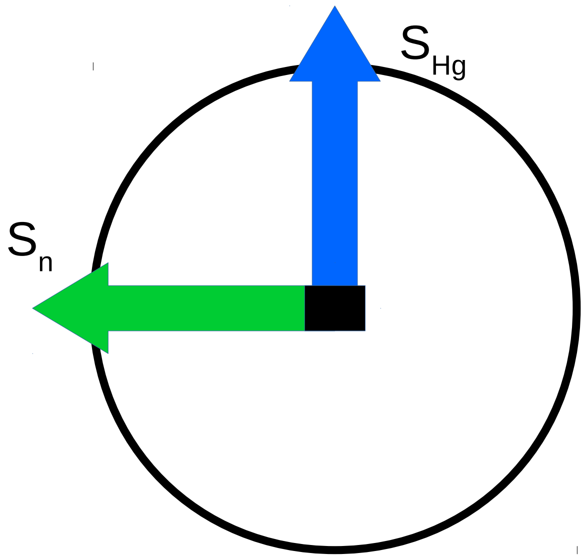
Mercury
Scattering Length Measurement

Motivation

Pseudo-magnetic field from **spin** dependent **Strong** interaction

$$B^* = \frac{-4\pi\hbar}{m_n \gamma_n} \rho b_i P \sqrt{\frac{I}{I+1}}$$



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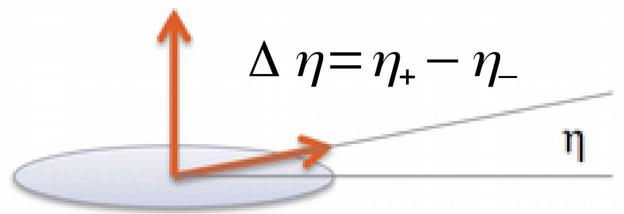
- Potential systematic false effect

$$d_n^{false} = \hbar \frac{\gamma_n}{4E} B^* \Delta\eta$$



- Calibration measurement for the nEDM apparatus

- Sign of incoherent scattering length

Neutron scattering lengths and cross sections							
Isotope	conc	Coh b	Inc b	Coh xs	Inc xs	Scatt xs	Abs xs
Hg	---	12.692	---	20.24	6.6	26.8	372.3(4.0)
196Hg	0.2	30.3(1.0)	0	115.(8.)	0	115.(8.)	3080.(180.)
198Hg	10.1	---	0	---	0	---	2
199Hg	17	16.9	(+/-)15.5	36.(2.)	30.(3.)	66.(2.)	2150.(48.)
200Hg	23.1	---	0	---	0	---	<60.
201Hg	13.2	---	---	---	---	---	7.8(2.0)
202Hg	29.6	---	0	---	0	9.828	4.89
204Hg	6.8	---	0	---	0	---	0.43

b_i Measurement

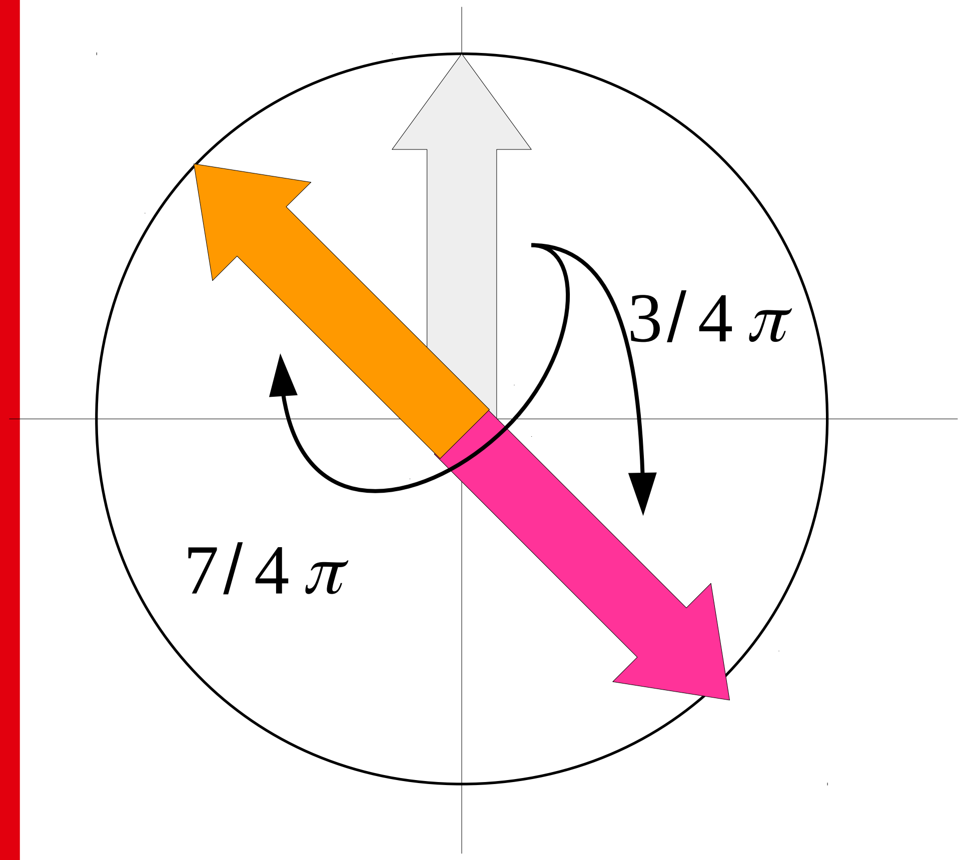
Frequency shift due to incoherent scattering length

$$b_i = \frac{-\gamma_n}{2.49\times 10^{-15}\,Tm^2} \times \frac{\partial \Delta f_n}{\partial\left(\rho_{Hg} P_{Hg}\right)}$$

Mercury incoherent scattering length

Hg Density and Hg polarisation along B_0

Mercury flipping pulse



Disadvantage:

ReduceHg-polarisation and effect by factor √2

Advantage:

Still able to monitor magnetic field using Hg magnetometer

Data in Batches

Batch	Temperature	Hg pulse
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Software crash then ramp up T

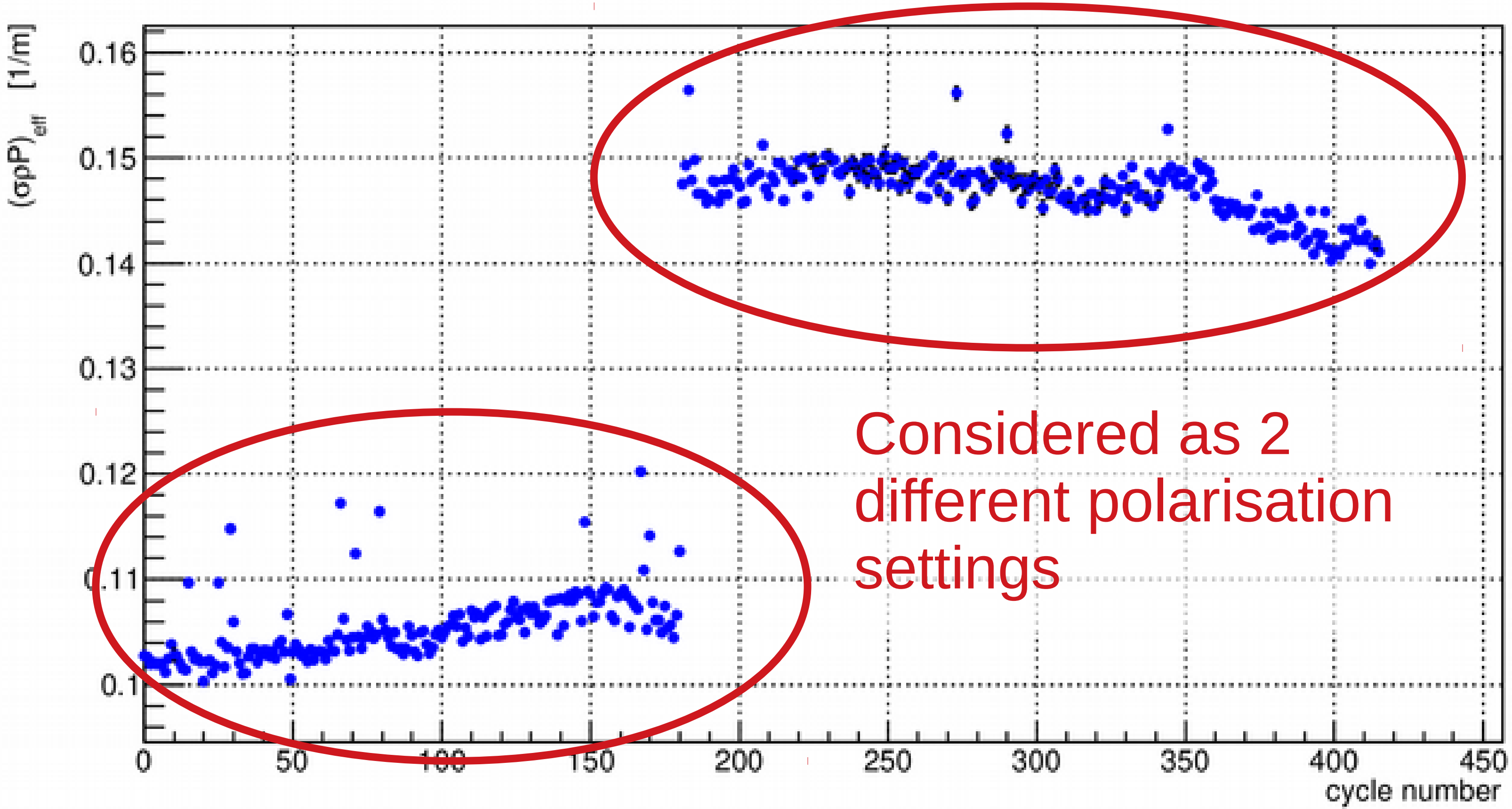
Have some influence on density and polarisation

3/4π flipping time

7/4π flipping time

Sequence: ABBABAAB

Special Case Batch1



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Frequency shift due to incoherent scattering length

Neutron frequency

__Data point:__

$$\Delta\nu = f_{Hg}\left|\frac{\gamma_n}{\gamma_{Hg}}\right| - f_{RF} \qquad \mathscr{A} = \frac{N_+ - N_-}{N_+ + N_-}$$

__*Notations*__

f_n: neutron real frequency
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N_+: neutron with spin up
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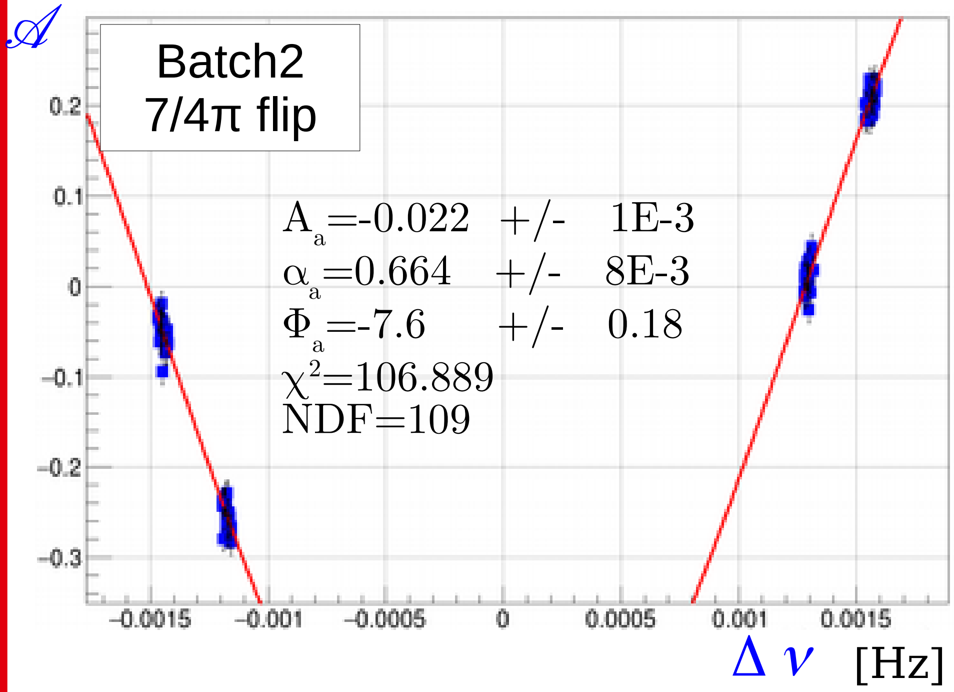

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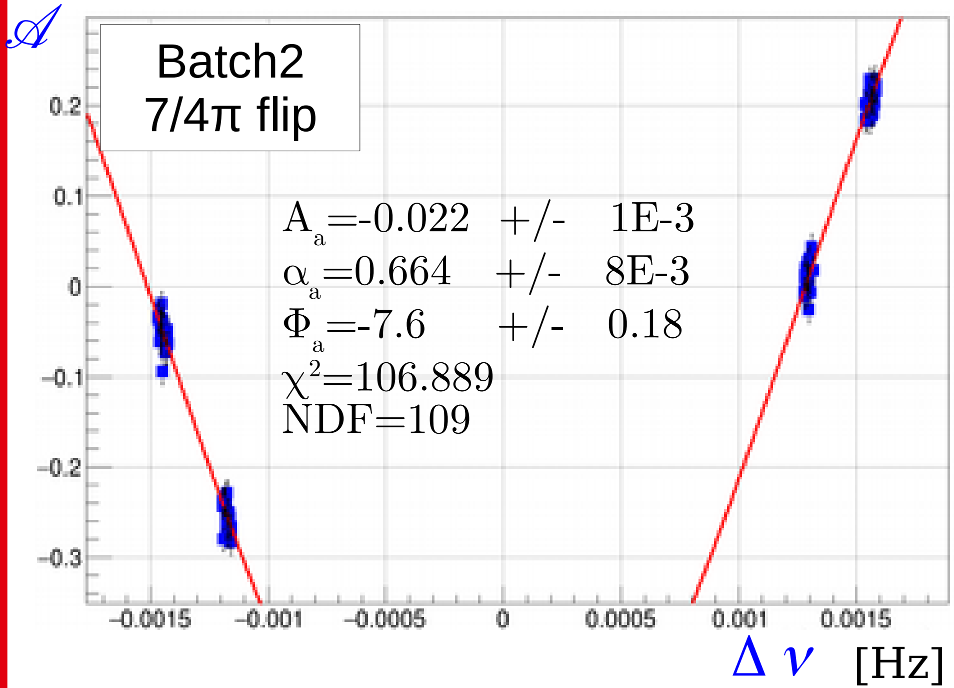

Fit formula:

$$\mathscr{A} = A_a - |\alpha_a| \cos\left(2\pi T\left(\Delta\nu + \Delta f_n\right)\right)$$

In red, parameters of the fit function (identical for each point with same configuration)
In blue, values assigned to each data point

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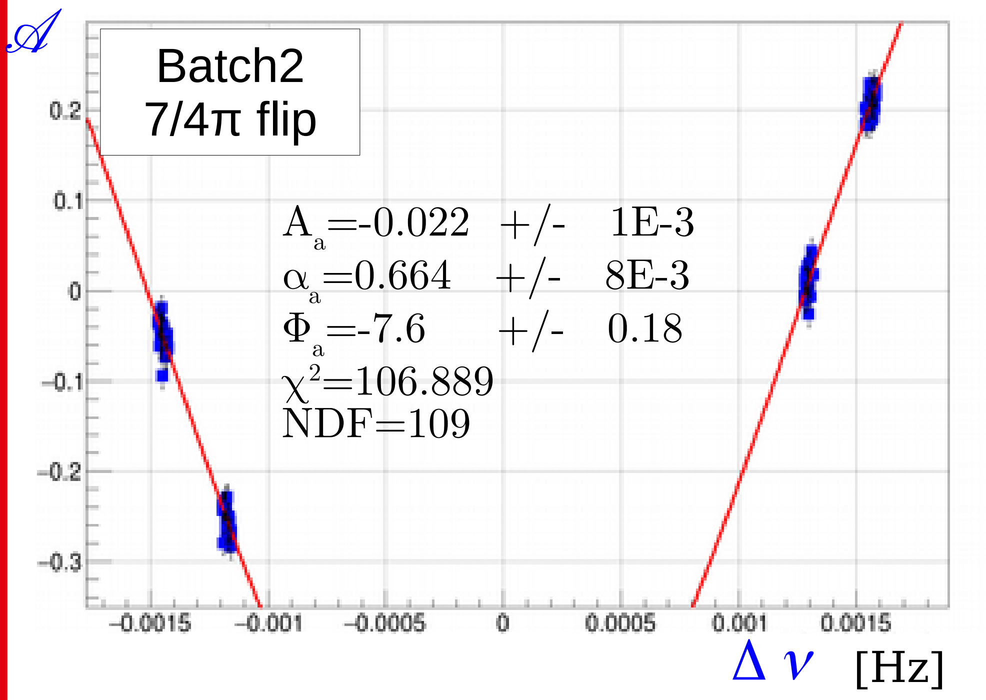

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By reversing the fit formula for ***each individual point***:

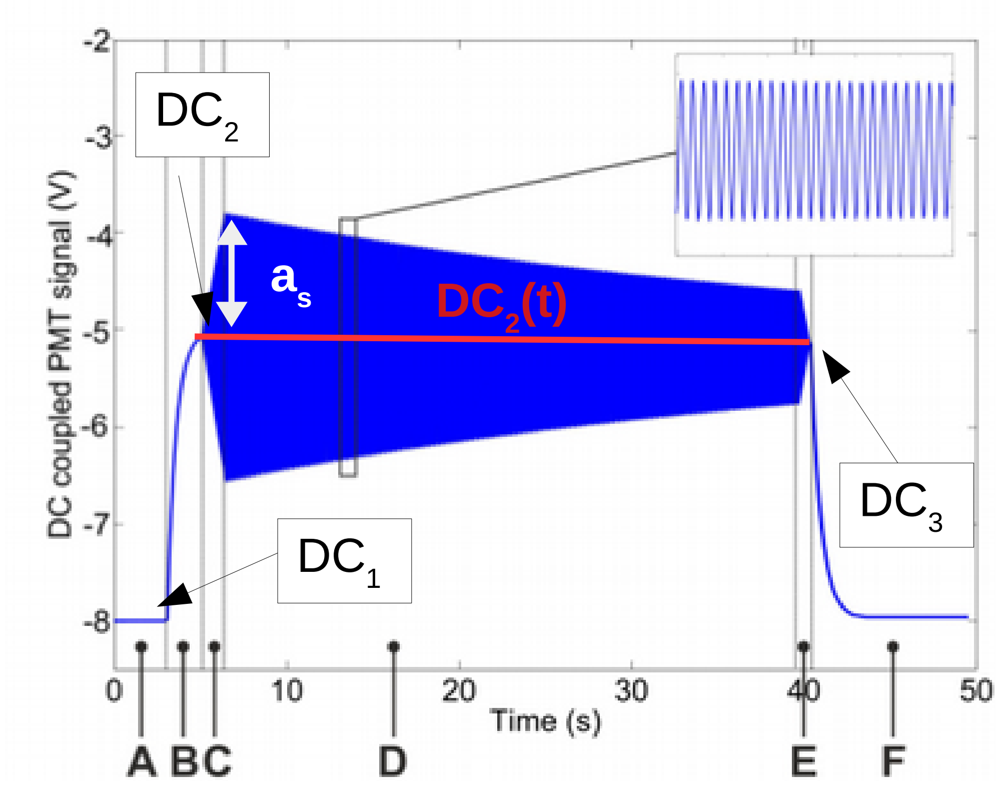
$$\Delta f_n = \frac{sgn(\Delta \nu)}{2\pi T} acos\left(\frac{A_a - \mathscr{A}}{|\alpha_a|}\right) - \Delta \nu$$

In red, parameters of the fit function (identical for each point with same configuration)
In blue, values assigned to each data point

$$b_i = \frac{-\gamma_n}{2.49\times 10^{-15}\,Tm^2} \times \frac{\partial \Delta f_n}{\partial\left(\rho_{Hg} P_{Hg}\right)}$$

Hg Density and Hg polarisation along B_0

Density and Polarisation



From Sybille's Thesis

$$P = \frac{-\alpha\beta}{\ln\left(1-\frac{DC_1-DC_2}{DC_1-DC_{offset}}\right)} arcsh\left(\frac{a_s}{DC_2-DC_{Offset}}\right)$$

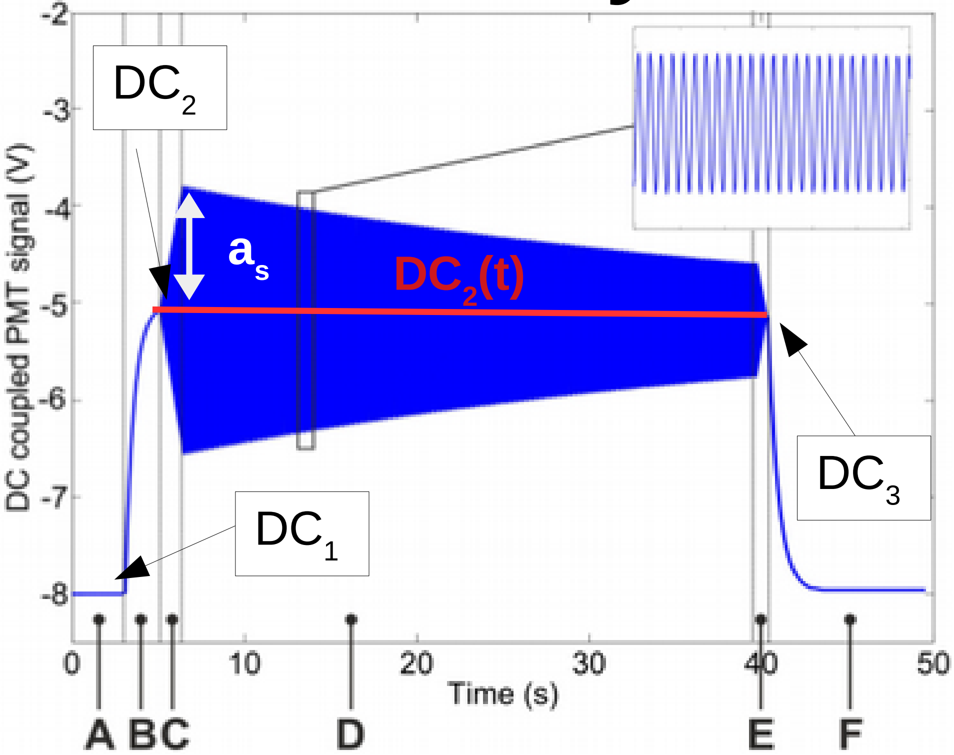
×

$$\rho.\sigma = \frac{-1}{L\beta}\ln\left(1-\frac{DC_1-DC_2}{DC_1-DC_{Offset}}\right)$$

=

$$\sigma\rho P = \frac{\alpha}{L} arcsh\left(\frac{a_s}{DC_2-DC_{Offset}}\right)$$

Density and Polarisation



Assumptions:

α=1

DC_{Offset}=0

L=0.47m

T : interaction time

τ : depolarisation time ($=T_1=T_2$)

T_3 : leakage time

Exponential decay

$$a_s(t) = as(0)\exp(-t/\tau)$$

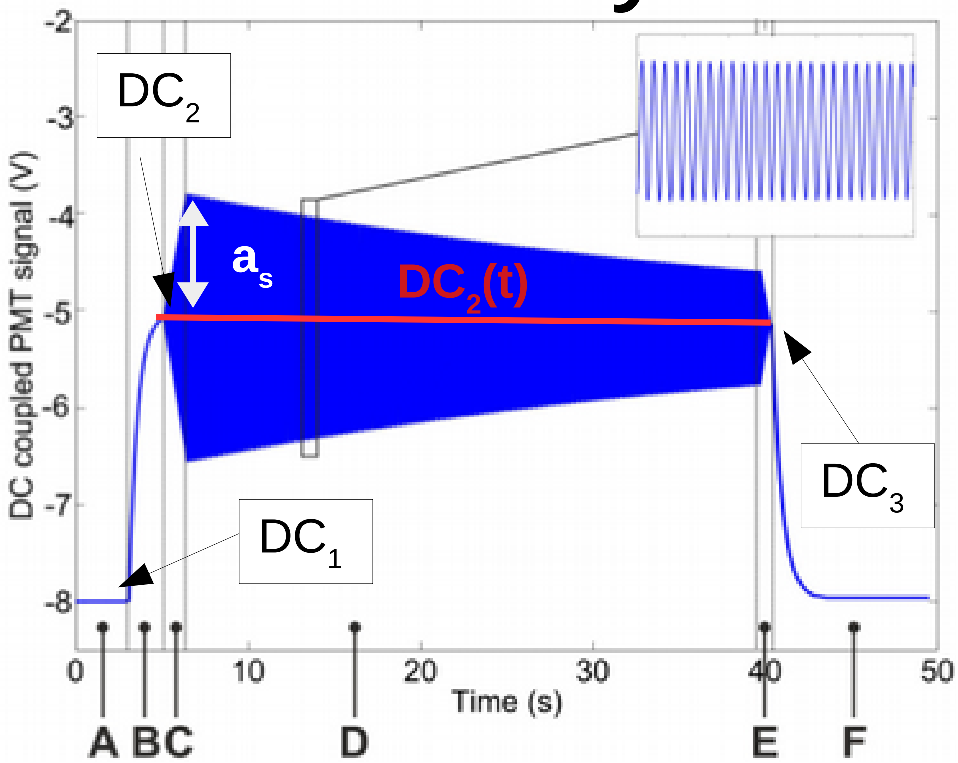
$$DC_2(t) = (DC_2(0) - DC_1)\exp(-t/T_3) + DC_1$$

Taylor expension as(t)<<DC2(t)

arcsh(x) ~x

$$\sigma \rho P = \frac{\alpha}{L} arcsh\left(\frac{a_s}{DC_2 - DC_{Offset}}\right)$$

Density and Polarisation



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Taylor expension as(t)<<DC2(t)
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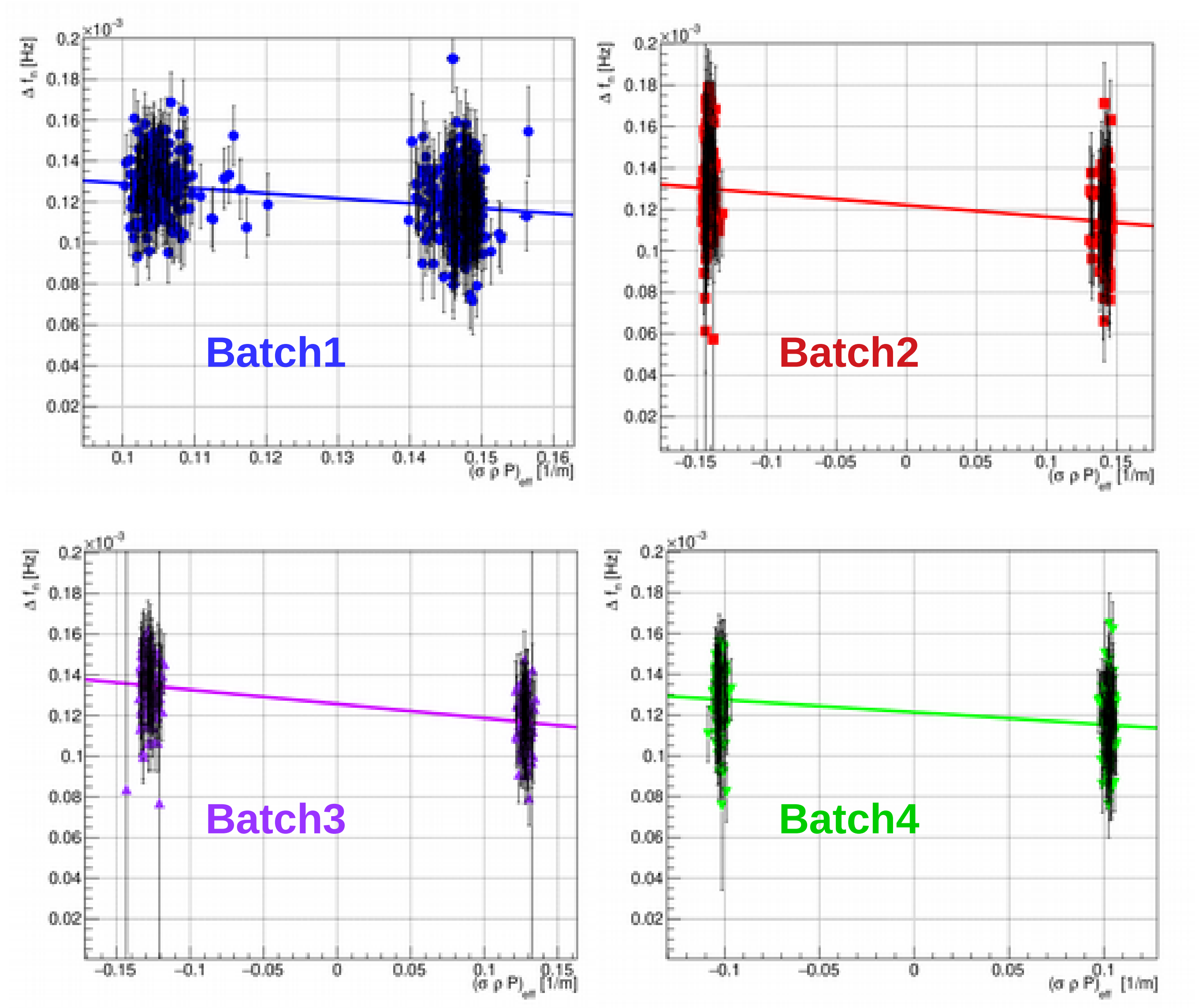
$$\sigma \rho P = \frac{\alpha}{L} arcsh\left(\frac{a_s}{DC_2 - DC_{Offset}}\right)$$

$$(\sigma \rho P)_{eff} = \frac{1}{LT}\int \frac{a_s(0)\exp(-t/\tau)}{(DC_2(0) - DC1)\exp(-t/T_3) + DC1} dt$$

Preliminary Results

$$b_i = \frac{-\gamma_n}{2.49\times10^{-15}\,Tm^2} \times \frac{\partial f_n}{\partial\left(\rho_{Hg} P_{Hg}\right)}$$

Δf_n vs. $(\sigma\rho P)_{eff}$



	Slope [Hz m]	Error [Hz m]
1	-24.4E-5	36.5E-6
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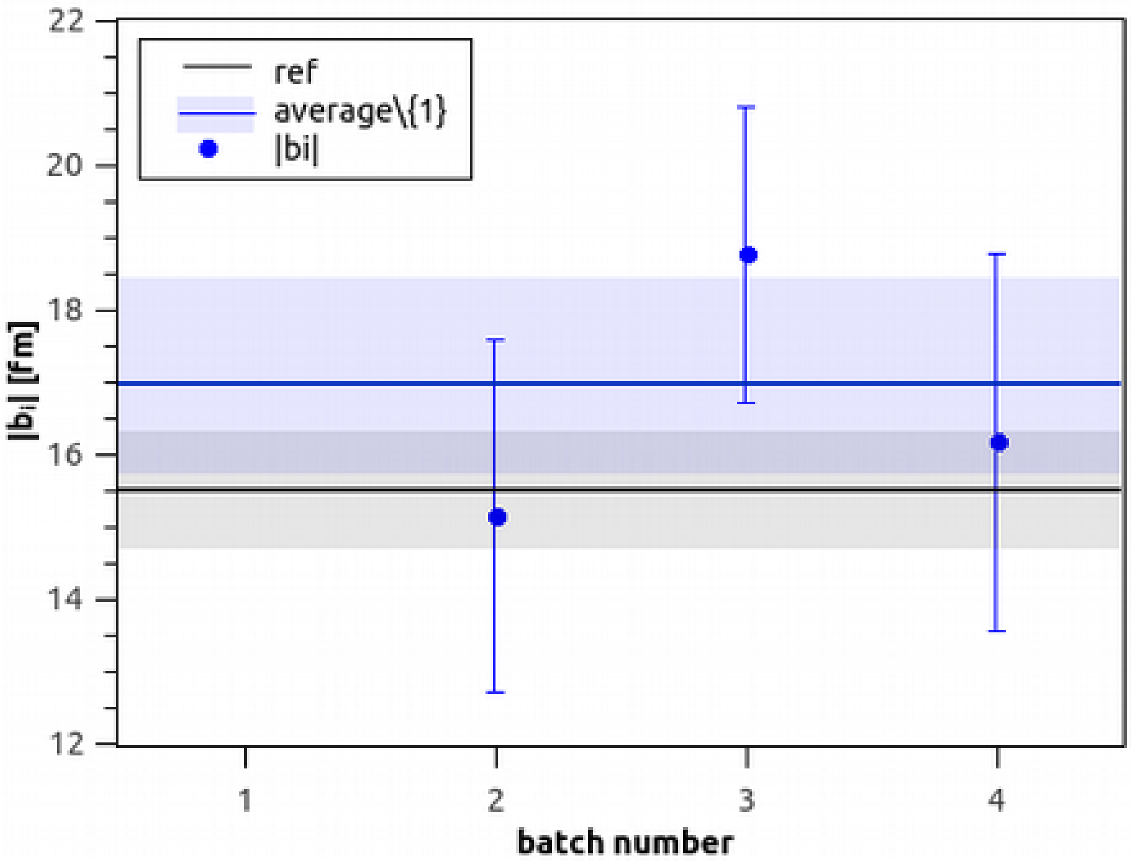
3	-6.70E-5	7.7E-6
4	-6.03E-5	9.70E-6

Absolute value of b_i



$$b_i = \frac{-\gamma_n \sigma}{2.49\times 10^{-15}\, Tm^2} \frac{\partial f_n}{\partial(\sigma \rho P)}$$

Our measurement

$|b_i|$=(16.98 +/- 1.35) fm

$|b_i|$=(17.88+/-1.33) fm

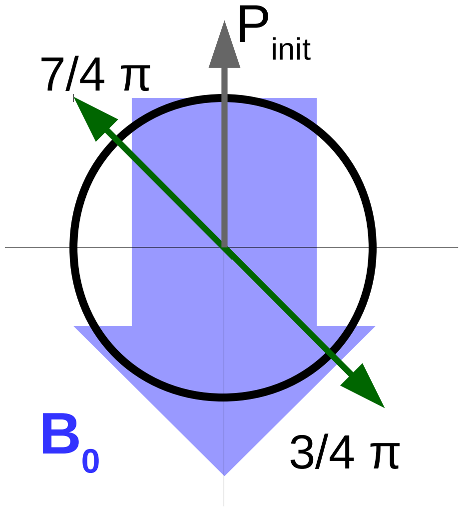
Standard value

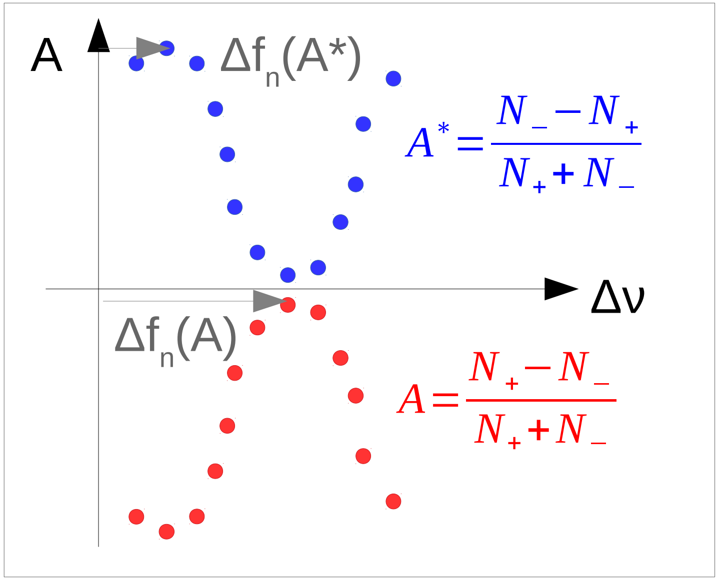
$|b_i|$=(15.5+/-0.8) fm

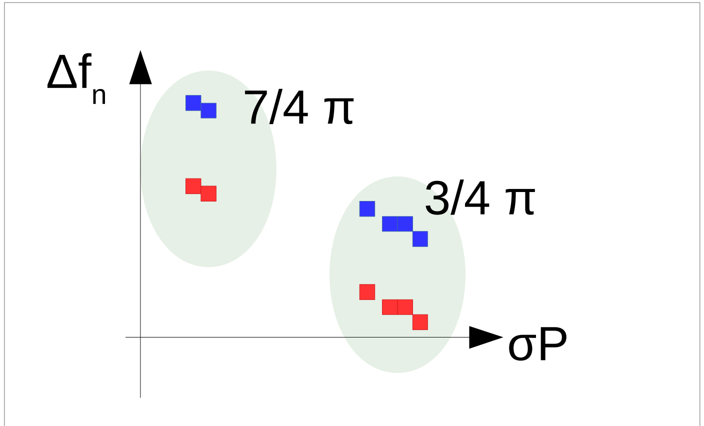
$\sigma = 1.95 \times 10^{-17}\ cm^2$ ***

*** theoretical value computed by with a factor ½ to take care of the circular polarisation

Sign determination







$$\frac{\delta}{\delta\rho P}(\Delta f_n(A)) = \frac{\delta}{\delta\rho P}(\Delta f_n(A^*))$$

$$b_i = \underbrace{\frac{-\gamma_n}{2.49\times10^{-15}}}_{>0} \times \underbrace{\frac{\partial \Delta f_n}{\partial(\rho_{Hg} P_{Hg})}}_{<0} < 0$$

Conclusion

- ✓ Preliminary result on bi (sign and value)
- ✓ Non zero measurement with the EDM apparatus

In addition

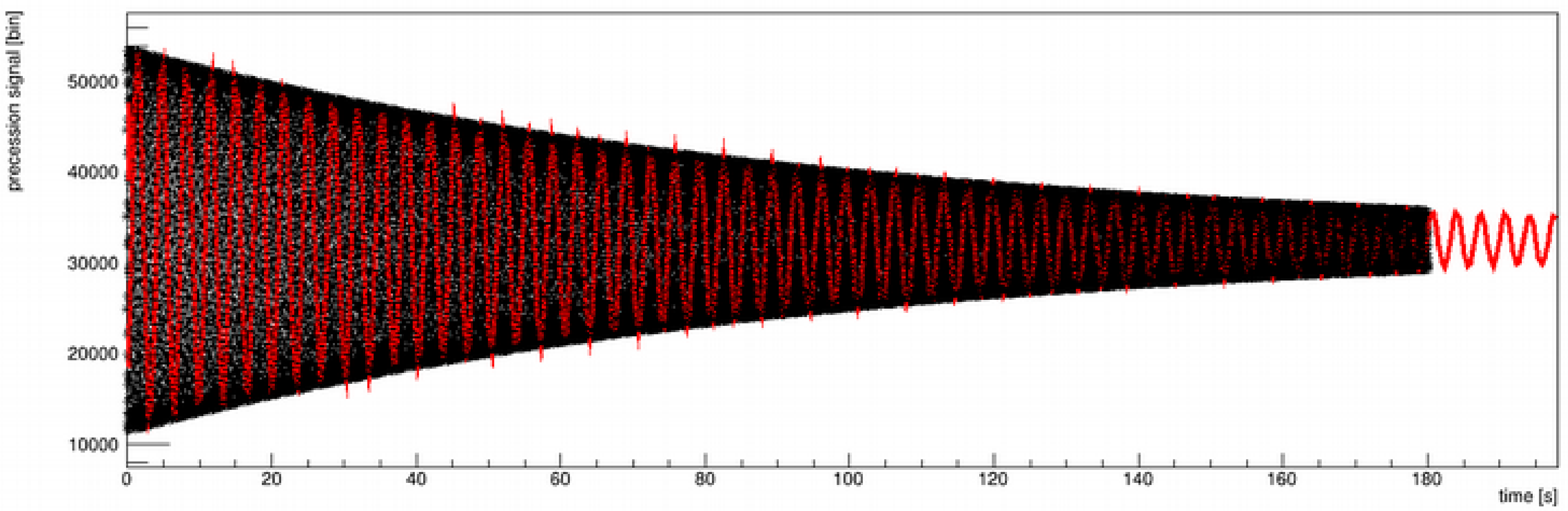
- ✓ Determination of the effect from each component on the result (especially the effect of T_3)
- ✓ Comparison of the values from metafiles with the values extracted from PMT and precession files

Will be cross check with second independent analysis

questions

Back up slides

τ, a_s from Precession or Meta file



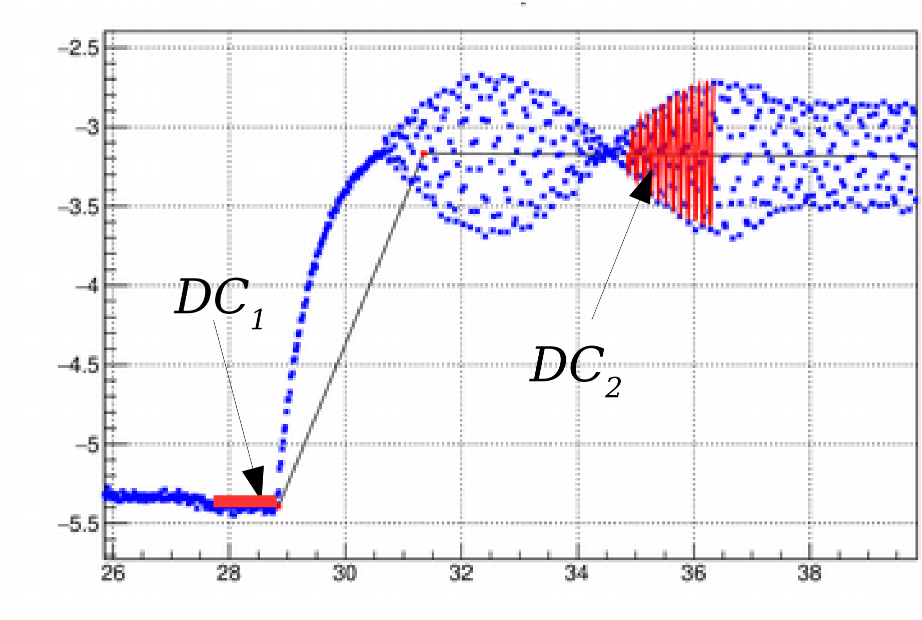
$$h(t)=|a_s|\cos(2\pi Bt-\phi)\times\exp(t/\tau)+C$$

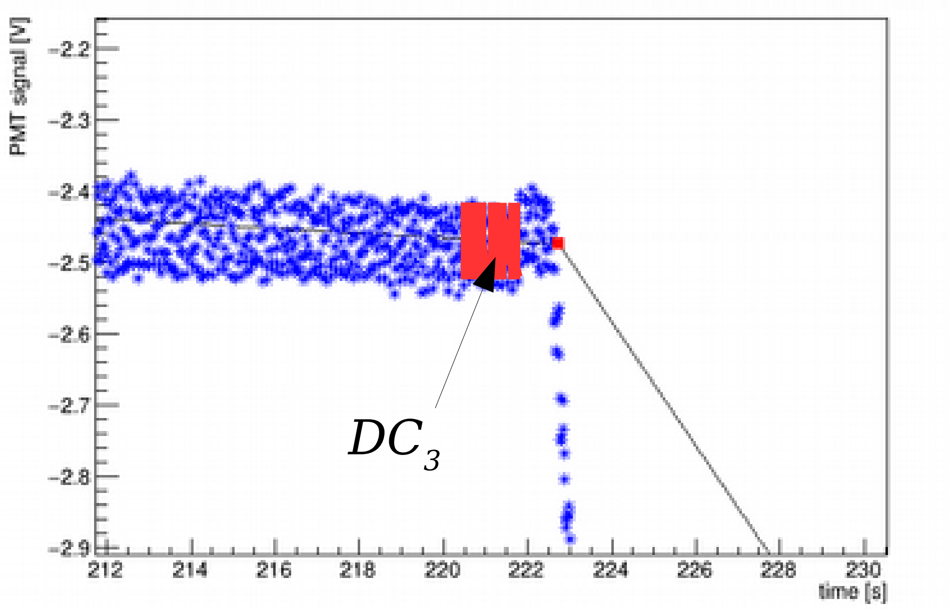
readNoise.C

DC1, DC2, DC3, T3 from PMT

$$T_3 = [t(DC_3) - t(DC_2)] \left[\ln \frac{(DC_2 - DC_1)}{(DC_3 - DC_1)} \right]^{-1}$$

Non zero correlation between T_3 and DC_1 and DC_2!





Error contributions

Mostly statistical

variable	value	error
Δf_n [Hz]	0.1 → 0.2	~0.015
$\lvert\sigma\rho P\rvert_{eff}$ [1/m]	0.1 → 0.15	<5E-4

Mostly from PMT extracted values

Error contributions

Variable	Value range	Associated error squared σ^2	Contribution on $\sigma^2(\sigma\rho P)$
DC_1	~ -6	~4E-3	~3E-10
DC_2	-3 → -0.5	4E-3 → 8E-3	1E-9 → 2E-7
T3	700 → 1000	5 → 30	1E-9 → 1E-7
τ	~110	5E-3	2E-12
a_s	0.2 → 0.4	7E-5 → 8E-5	5E-10 → 1E-9
cov(T3,DC1)			5E-11 → 1E-9
cov(T3,DC2)			5E-10 → 1E-7

Error calculation idea for $(\sigma\rho P)_{eff}$

$$y = f(a,b) = \int F(t,a,b)\,dt$$

Leibniz theorem

$$\sigma(y)^2 = \left(\frac{df}{da}\right)^2 \sigma(a)^2 + \left(\frac{df}{db}\right)^2 \sigma(b)^2 + 2\frac{df}{da}\frac{df}{db}\,cov(a,b)$$

$$= \left(\int \frac{dF}{da}dt\right)^2 \sigma(a)^2 + \left(\int \frac{dF}{db}dt\right)^2 \sigma(b)^2 + 2\left(\int \frac{dF}{da}dt\right)\left(\int \frac{dF}{db}dt\right) cov(a,b)$$

Special case for covariance factor

$$Cov(a, f(a)) = \frac{da}{da}\frac{df}{da}\sigma(a)^2$$

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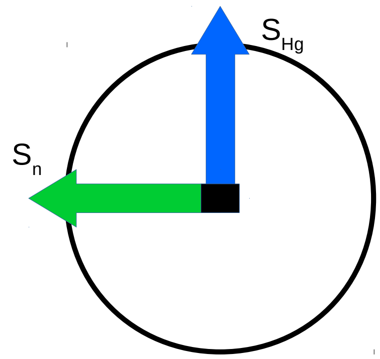

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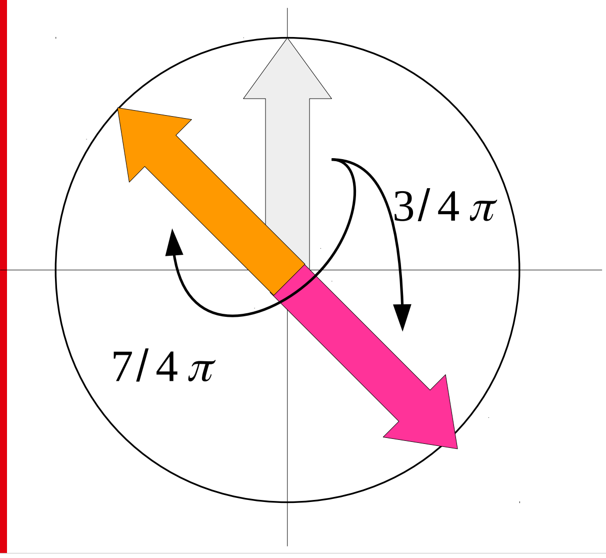

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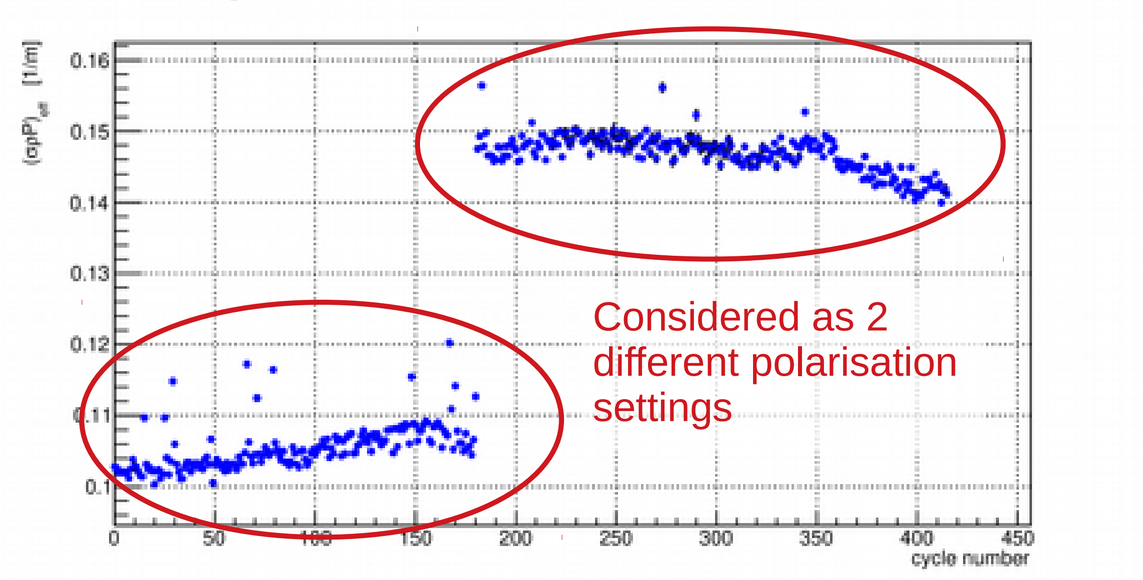
Sequence: ABBABAAB

Special Case Batch1



For all of them we observe a drift of at least 0.01 m-1
Except batch4 wich seems slightly more stable

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$\mathscr{A}$

Batch2
7/4π flip

A_a=-0.022 +/- 1E-3
α_a=0.664 +/- 8E-3
Φ_a=-7.6 +/- 0.18
χ^2=106.889
NDF=109

$\Delta\,\nu$ [Hz]

Neutron frequency

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$$\mathscr{A} = A_a - |\alpha_a| \cos(2\pi T(\Delta\nu + \Delta f_n))$$

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By reversing the fit formula for ***each individual point***:

$$\Delta f_n = \frac{sgn(\Delta\nu)}{2\pi T} acos\left(\frac{A_a - \mathscr{A}}{|\alpha_a|}\right) - \Delta\nu$$

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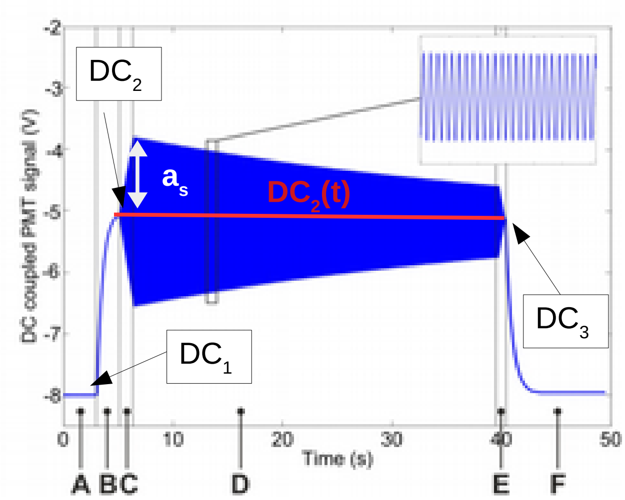
Density and Polarisation



From Sybille's Thesis

$$P=\frac{-\alpha\beta}{\ln\left(1-\frac{DC_1-DC_2}{DC_1-DC_{offset}}\right)}arcsh\left(\frac{a_s}{DC_2-DC_{Offset}}\right)$$

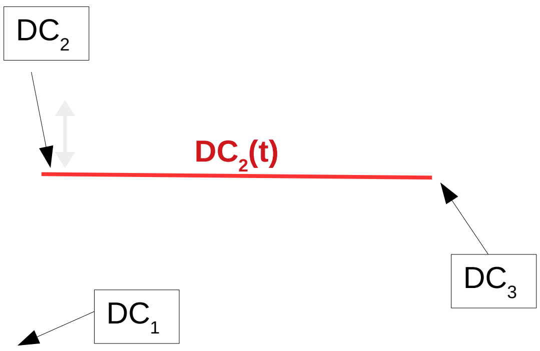
$$\times$$

$$\rho.\sigma=\frac{-1}{L\beta}\ln\left(1-\frac{DC_1-DC_2}{DC_1-DC_{Offset}}\right)$$

$$=$$

$$\sigma\rho P=\frac{\alpha}{L}arcsh\left(\frac{a_s}{DC_2-DC_{Offset}}\right)$$

Density and Polarisation



Assumptions:

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Exponential decay

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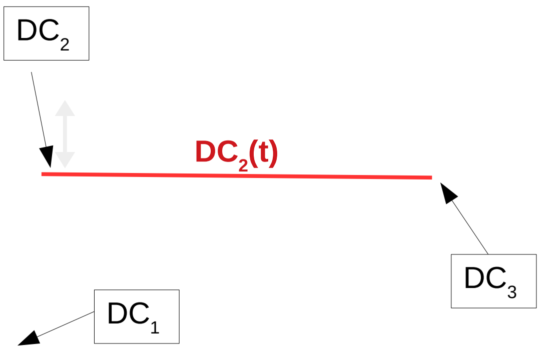
Density and Polarisation



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$$(\sigma\rho P)_{eff} = \frac{1}{LT}\int \frac{a_s(0)\exp(-t/\tau)}{(DC_2(0) - DC1)\exp(-t/T_3) + DC1}\, dt$$

Preliminary Results

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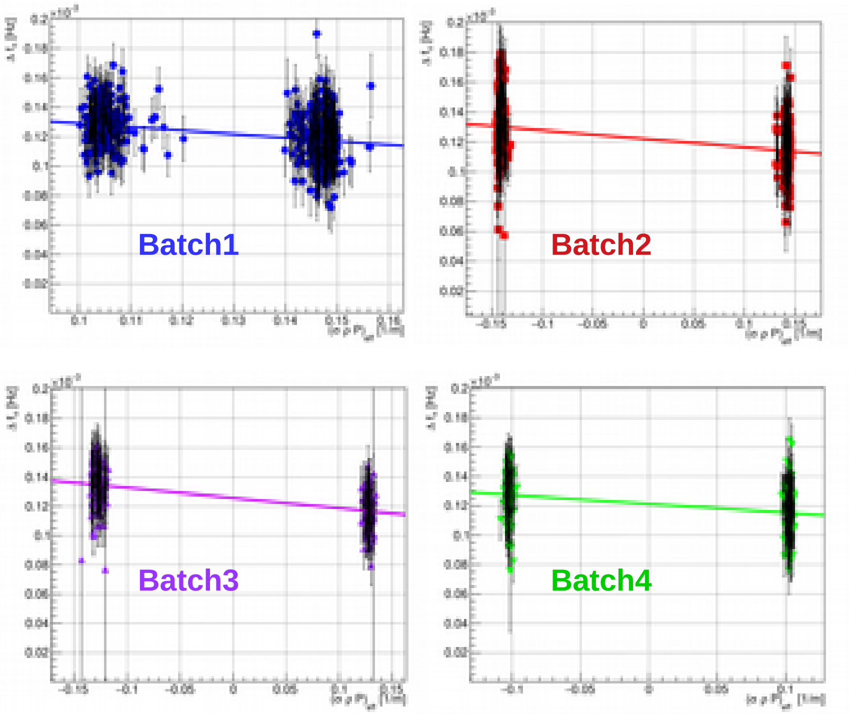
28/11/2019

Δf_n vs. $(\sigma\rho P)_{eff}$



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Absolute value of b_i

$$b_i = \frac{-\gamma_n \sigma}{2.49 \times 10^{-15}\, Tm^2} \frac{\partial f_n}{\partial(\sigma \rho P)}$$

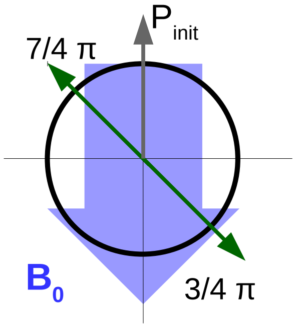
Our measurement
$|b_i|$=(16.98 +/- 1.35) fm
$|b_i|$=(17.88+/-1.33) fm

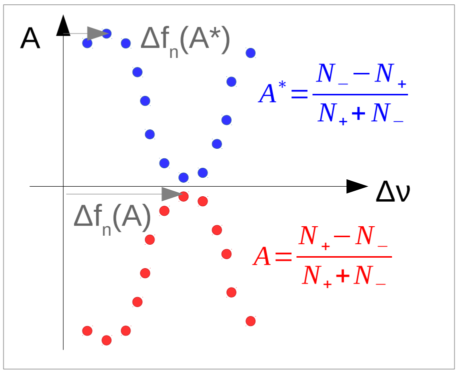
Standard value
$|b_i|$=(15.5+/-0.8) fm

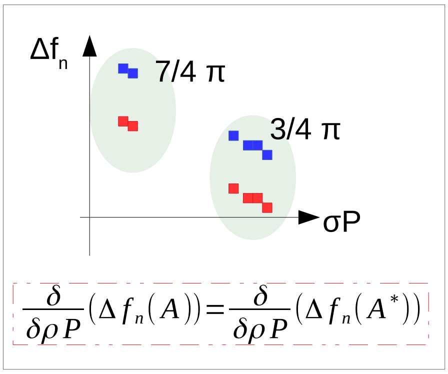
$\sigma = 1.95 \times 10^{-17}$ cm^2 ***

*** theoretical value computed by with a factor ½ to take care of the circular polarisation

Sign determination







$$b_i = \underbrace{\frac{-\gamma_n}{2.49\times10^{-15}}}_{>0} \times \underbrace{\frac{\partial\,\Delta f_n}{\partial(\rho_{Hg}P_{Hg})}}_{<0} < 0$$

Conclusion

- Preliminary result on bi (sign and value)
- Non zero measurement with the EDM apparatus

In addition

- Determination of the effect from each component on the result (especially the effect of T_3)
- Comparison of the values from metafiles with the values extracted from PMT and precession files

Will be cross check with second independent analysis

questions ?

Back up slides

τ, a_s from Precession or Meta file

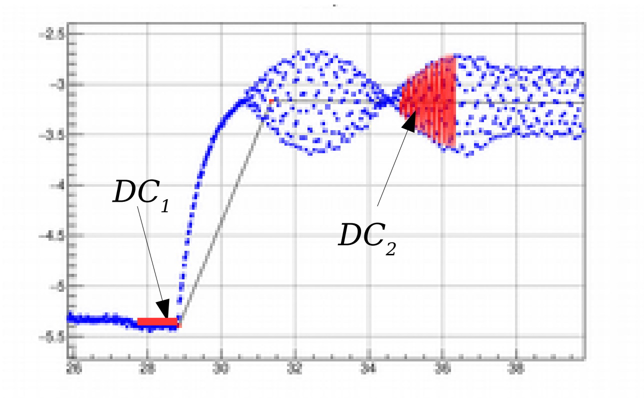
$$h(t)=|a_s|\cos(2\pi Bt-\phi)\times\exp(t/\tau)+C$$

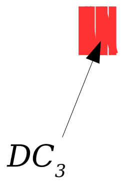
readNoise.C

DC1, DC2, DC3, T3 from PMT

$$T_3=[t(DC_3)-t(DC_2)]\left[\ln\frac{(DC_2-DC_1)}{(DC_3-DC_1)}\right]^{-1}$$

Non zero correlation between T_3 and DC_1 and DC_2!





Error contributions

variable	value	error
Δf_n [Hz]	0.1 → 0.2	~0.015
$\|\sigma\rho P\|_{eff}$ [1/m]	0.1 → 0.15	<5E-4

Mostly statistical

Mostly from PMT extracted values

Error contributions

Variable	Value range	Associated error squared σ^2	Contribution on $\sigma^2(\sigma\rho P)$
DC_1	~ -6	~4E-3	~3E-10
DC_2	-3 → -0.5	4E-3 → 8E-3	1E-9 → 2E-7
T3	700 → 1000	5→ 30	1E-9→ 1E-7
τ	~110	5E-3	2E-12
a_s	0.2→ 0.4	7E-5 → 8E-5	5E-10 → 1E-9
cov(T3,DC1)			5E-11 → 1E-9
cov(T3,DC2)			5E-10→ 1E-7

Error calculation idea for $(\sigma\rho P)_{eff}$

$$y = f(a,b) = \int F(t,a,b)\,dt$$

Leibniz theorem

$$\sigma(y)^2 = \left(\frac{df}{da}\right)^2 \sigma(a)^2 + \left(\frac{df}{db}\right)^2 \sigma(b)^2 + 2\frac{df}{da}\frac{df}{db}\,cov(a,b)$$

$$= \left(\int \frac{dF}{da}dt\right)^2 \sigma(a)^2 + \left(\int \frac{dF}{db}dt\right)^2 \sigma(b)^2 + 2\left(\int \frac{dF}{da}dt\right)\left(\int \frac{dF}{db}dt\right) cov(a,b)$$

Special case for covariance factor

$$Cov(a, f(a)) = \frac{da}{da}\frac{df}{da}\sigma(a)^2$$