



Mercury

Scattering length measurement

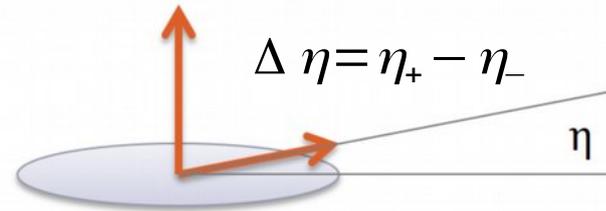
# Motivation

Pseudo-magnetic field from **spin** dependent **Strong** interaction

$$B^* = \frac{-4\pi\hbar}{m_n \gamma_n} \rho b_i P \sqrt{\frac{I}{I+1}}$$

- Potential systematic false effect

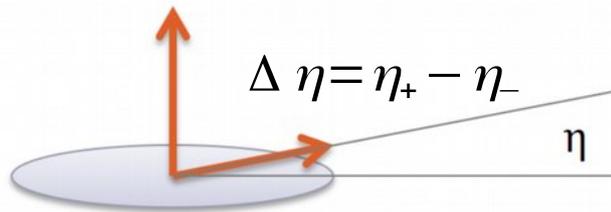
$$d_n^{false} = \hbar \frac{\gamma_n}{4E} B^* \Delta \eta$$



- Calibration measurement for the nEDM apparatus
- Sign of incoherent scattering length

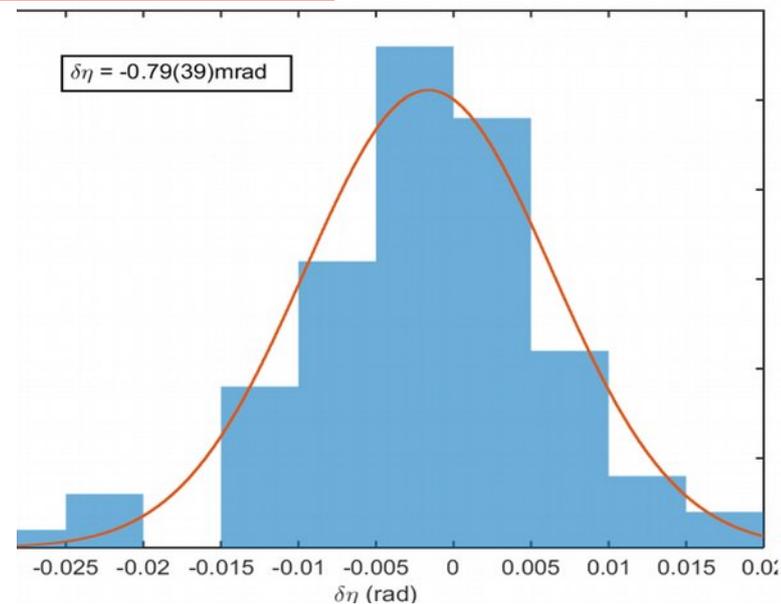
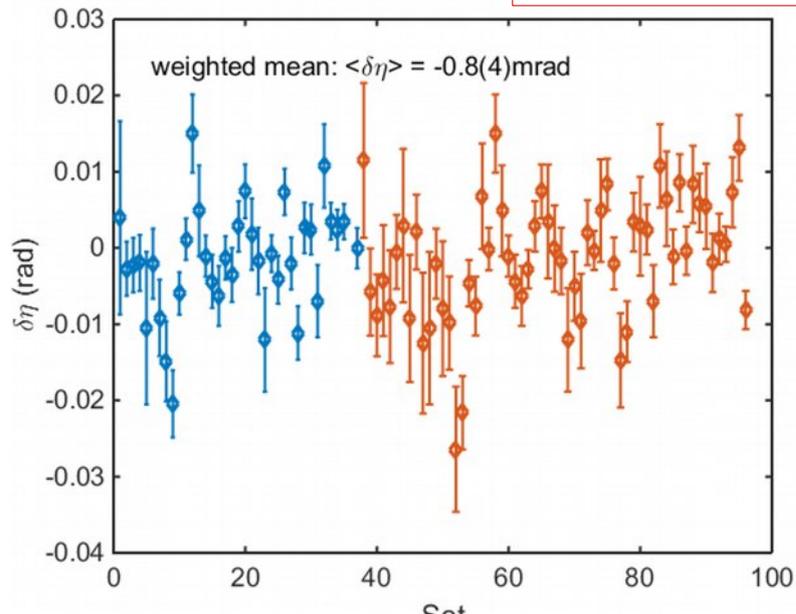
Neutron scattering lengths and cross sections							
Isotope	conc	Coh b	Inc b	Coh xs	Inc xs	Scatt xs	Abs xs
Hg	---	12.692	---	20.24	6.6	26.8	372.3(4.0)
196Hg	0.2	30.3(1.0)	0	115.(8.)	0	115.(8.)	3080.(180.)
198Hg	10.1	---	0	---	0	---	2
199Hg	17	16.9	(+/-)15.5	36.(2.)	30.(3.)	66.(2.)	2150.(48.)
200Hg	23.1	---	0	---	0	---	<60.
201Hg	13.2	---	---	---	---	---	7.8(2.0)
202Hg	29.6	---	0	---	0	9.828	4.89
204Hg	6.8	---	0	---	0	---	0.43

# Systematic effect



$$d_n^{false} = \hbar \frac{\gamma_n}{4 E} B^* \Delta \eta$$

$$d_n^{false} = \pm 2.1 (4.5) 10^{-28} \text{ ecm}$$

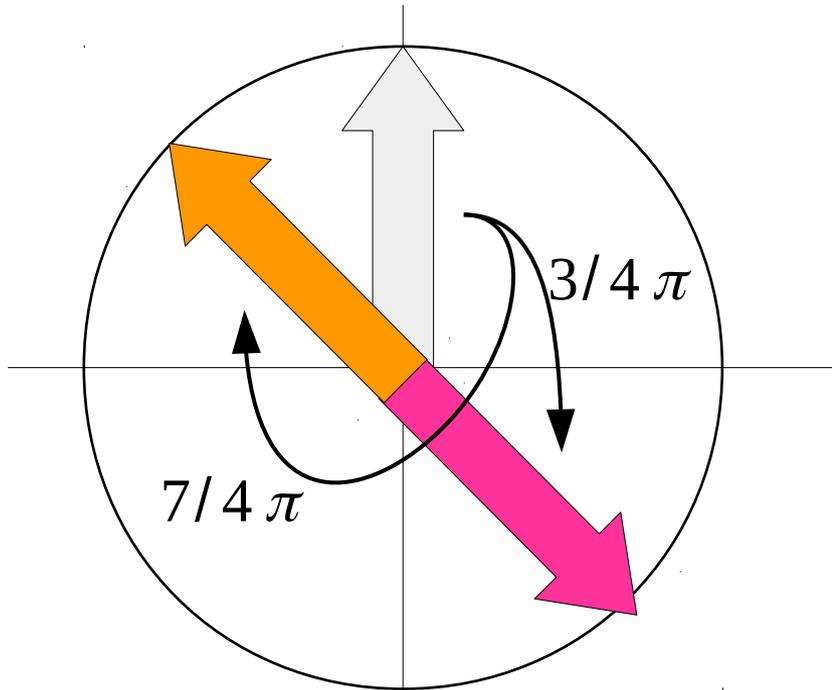


(Phillip's  
talk Bern)

# $b_i$ measurement

Disadvantage: Reduce Hg-polarisation/effect by factor  $\sqrt{2}$

Advantage: Still able to monitor magnetic field using Hg magnetometer



Batch	Temperature	Hg pulse
1	200°C 225°C	1.5s / 1.5s 1.5s / 1.5s
2	225°C	1.5s / 3.5s
3	225°C	3s / 7s
4	210°C	3s / 7s

Sequence: ABBABAAB

# Formula

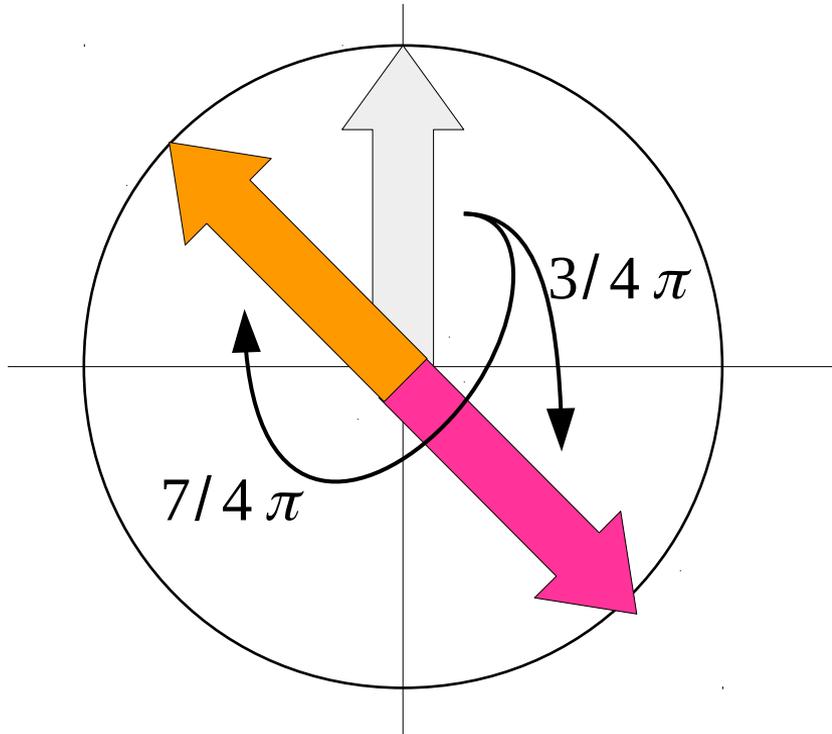
$$\phi^* = -\gamma_n B^*_{Hg} T$$

&

$$B^* = \frac{-4\pi\hbar}{m_n \gamma_n} \rho b_i P \sqrt{\frac{I}{I+1}}$$

$$\dots = 2.49 \times 10^{-15} \rho b_i P$$

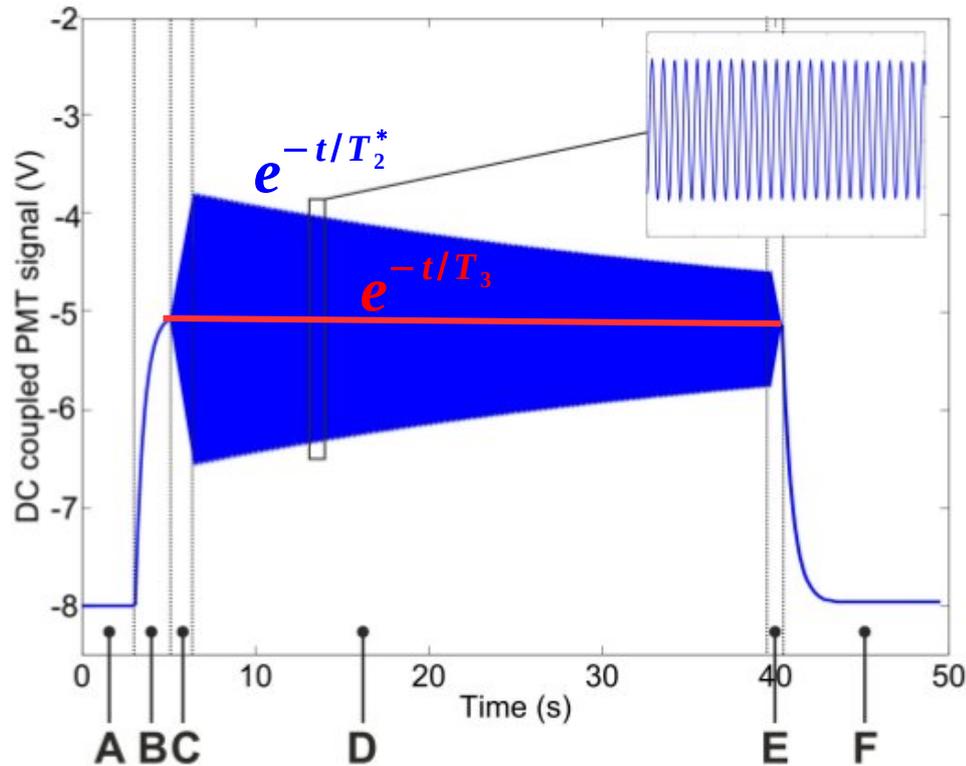
[Tm<sup>2</sup>]



$$b_i = \frac{-1}{\gamma_n T \times 2.49 \times 10^{-15}} \times \frac{\partial \phi_n}{\partial (\rho_{Hg} P_{Hg})}$$

$$(T = 182.5 \text{ s})$$

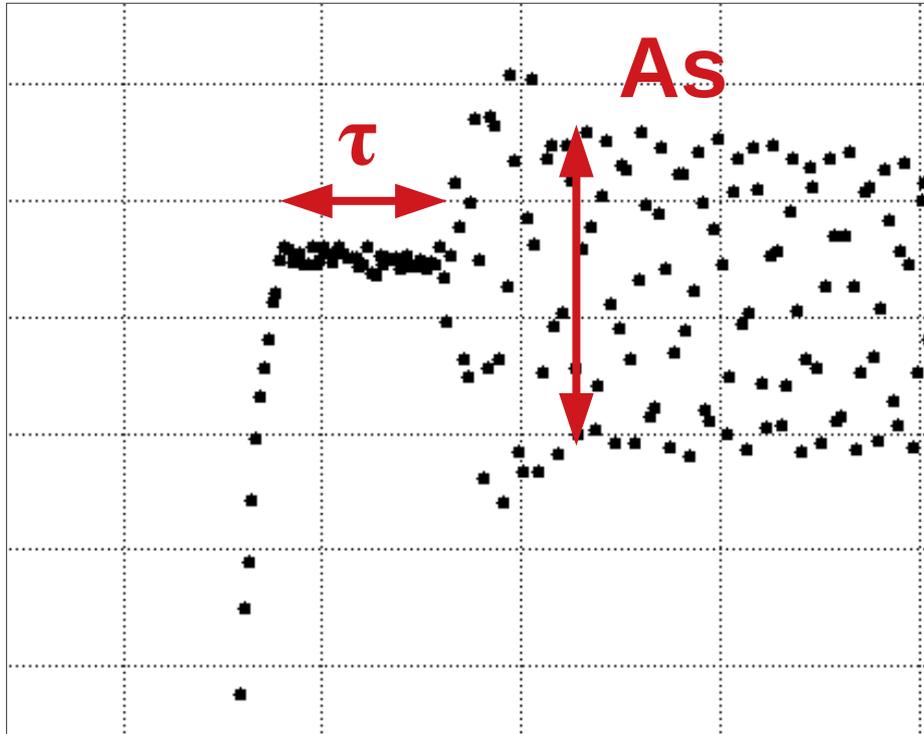
# Definition



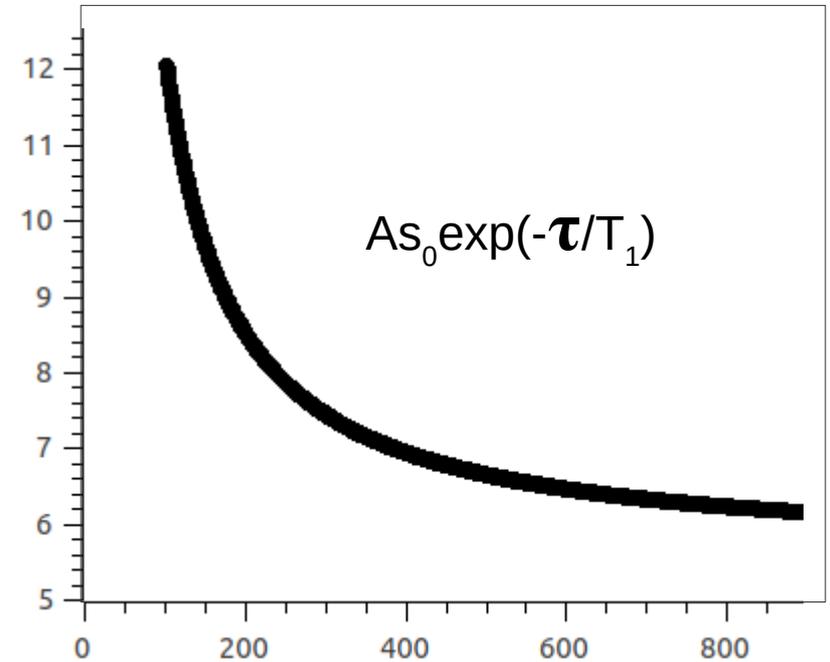
- T : interaction time  $\sim 182.5$ s
- $T_1$  : longitudinal depolarisation
- $T_2$  : transversale depolarisation
- $T_2^*$  : overall depolarisation  
 $\sim$ mix of  $T_1$  and  $T_2$
- $T_3$  : leak time
- $T^*$  : combinaison of  $T_1$  and  $T_3$   

$$1/T^* = 1/T_1 - 1/T_3$$

# $T_1$ measurement

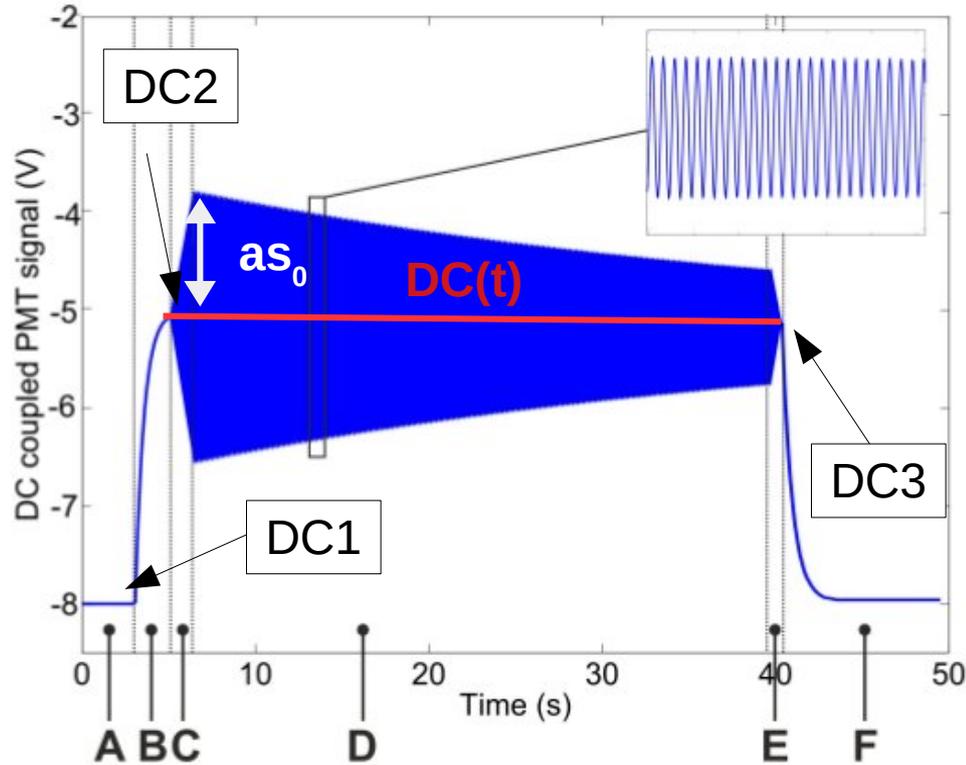


$As$



$\tau$

# Density Polarisation: $\rho P_{\text{eff}}$

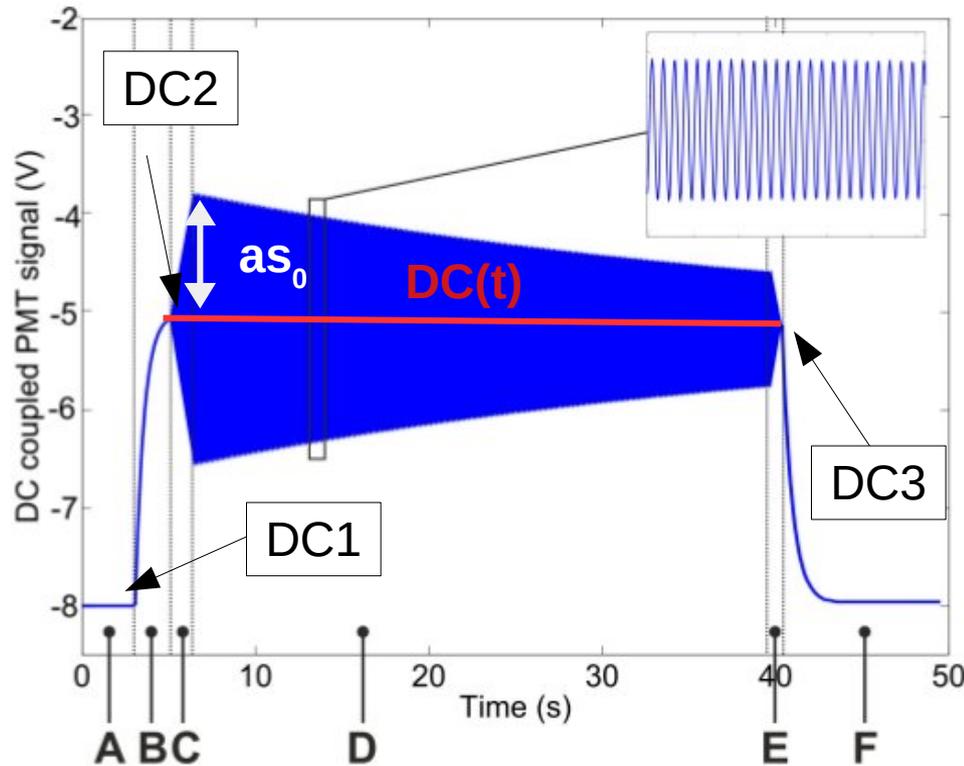


Generic formula:

$$\sigma \rho P = \frac{1}{L} a s h\left(\frac{as}{DC_2}\right)$$

$$L = 0.47 \text{ m}$$

# Density Polarisation: $\rho P_{\text{eff}}$



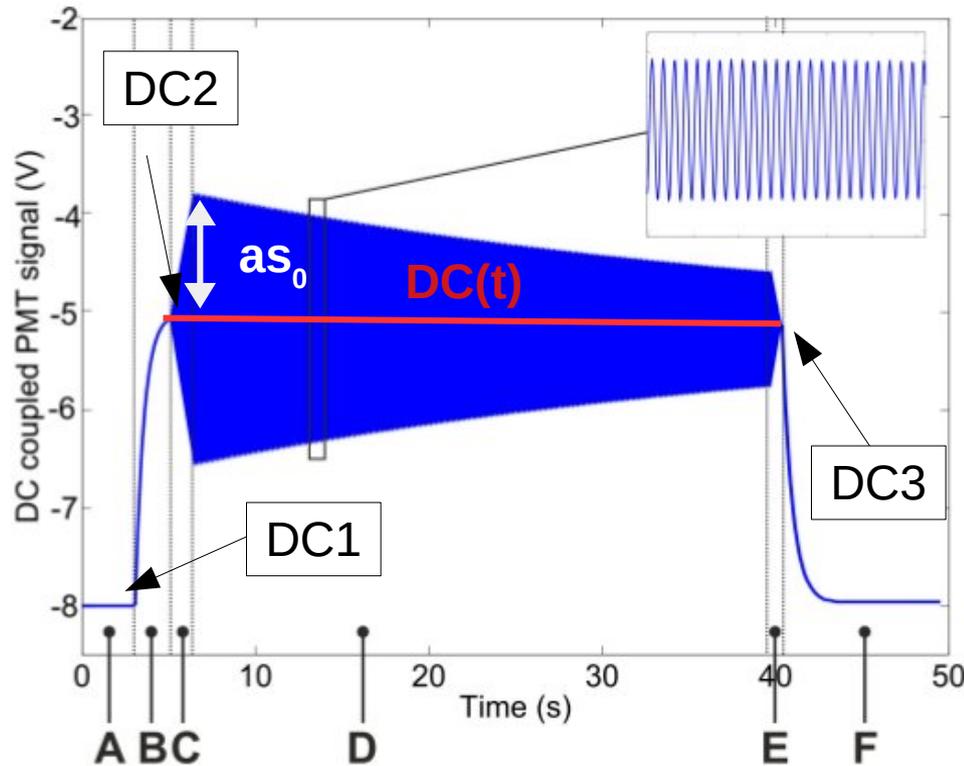
Generic formula:

$$\sigma \rho P = \frac{1}{L} a s h\left(\frac{a s}{DC_2}\right) \quad L = 0.47 \text{ m}$$

Effective value:

$$(\sigma \rho P)_{\text{eff}} = \frac{1}{LT} \int a s h\left(\frac{As(t)}{DC(t)}\right) dt$$

# Density Polarisation: $\rho P_{\text{eff}}$



Generic formula:

$$\sigma \rho P = \frac{1}{L} \operatorname{ash}\left(\frac{as}{DC_2}\right) \quad L = 0.47 \text{ m}$$

Effective value:

$$(\sigma \rho P)_{\text{eff}} = \frac{1}{LT} \int \operatorname{ash} \frac{As(t)}{DC(t)} dt$$

Assumptions:

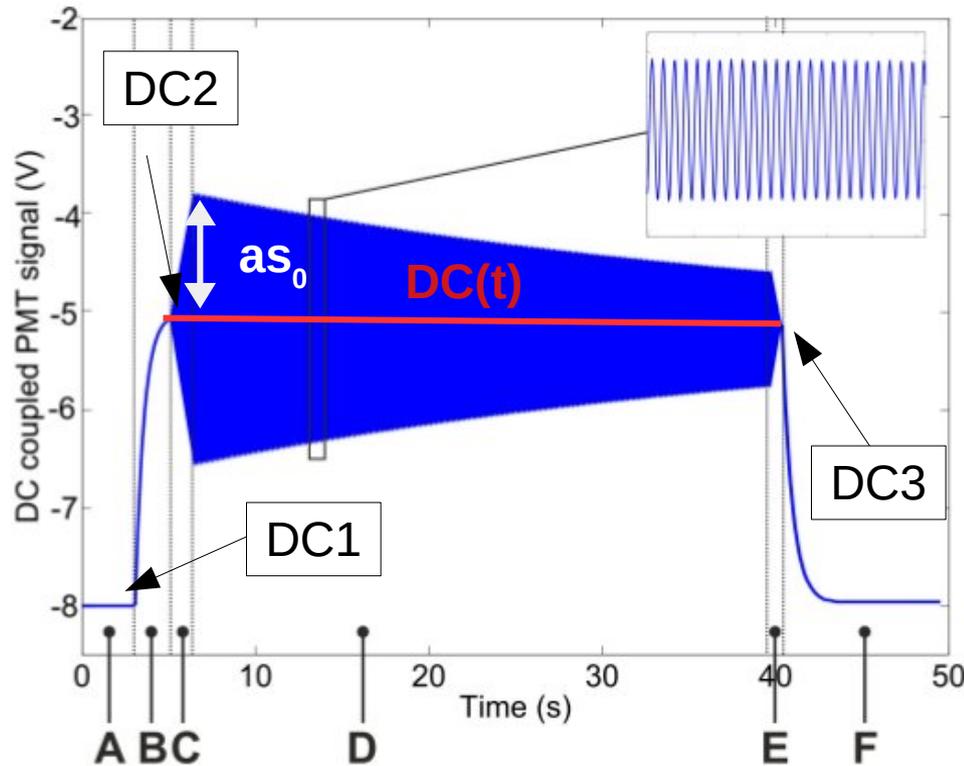
$$DC(t) = DC_2 \times e^{-t/T_3} \quad T_3 = T / \ln \frac{(DC_2 - DC_1)}{(DC_3 - DC_1)}$$

$$T = 182.5 \text{ s}$$

$$As(t) = as_0 \times e^{-t/T_1} \quad T_1 \sim 100 \text{ s}$$

$$T^{*-1} = 1/T_1 - 1/T_3$$

# Density Polarisation: $\rho P_{\text{eff}}$



Generic formula:

$$\sigma \rho P = \frac{1}{L} \text{ash}\left(\frac{as}{DC_2}\right) \quad L = 0.47 \text{ m}$$

Effective value:

$$(\sigma \rho P)_{\text{eff}} = \frac{1}{LT} \int \text{ash}\left(\frac{As(t)}{DC(t)}\right) dt$$

Assumptions:

$$\frac{As(t)}{DC(t)} = \frac{as_0}{DC_2} \times e^{-t/T^*}$$

$$T_3 = T / \ln\left(\frac{DC_2 - DC_1}{DC_3 - DC_1}\right)$$

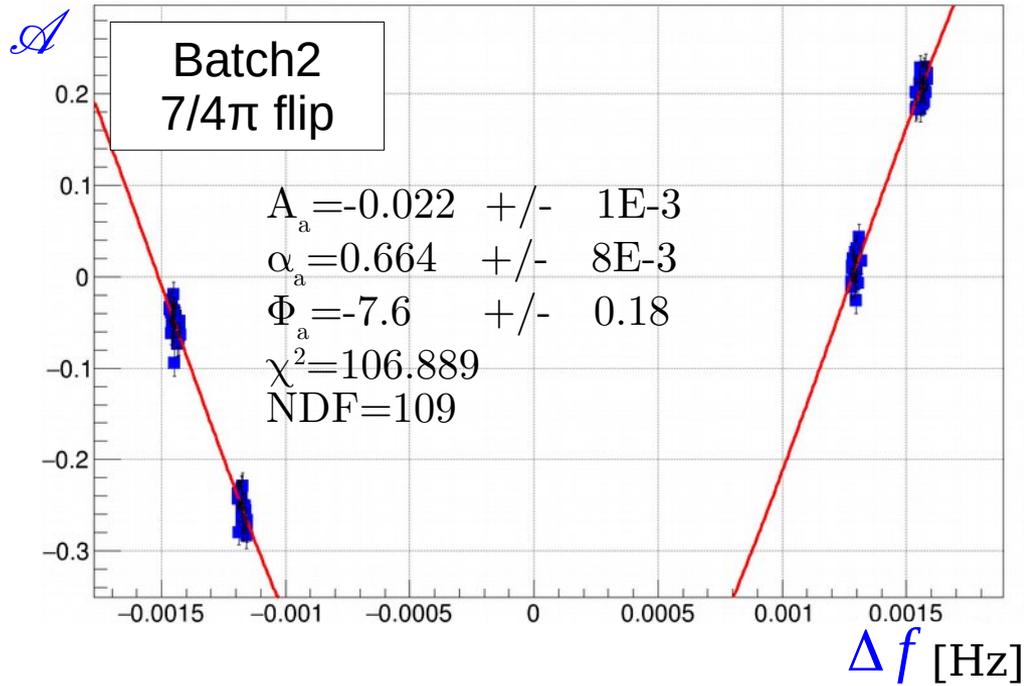
$$T = 182.5 \text{ s}$$

$$T_1 \sim 100 \text{ s}$$

$$T^{*-1} = 1/T_1 - 1/T_3$$

$$(\sigma \rho P)_{\text{eff}} \sim \frac{as_0 T^*}{DC_2 L T} (1 - e^{-T/T^*})$$

# Neutron phase $\varphi_n$



Data point:

$$\mathcal{A} = \frac{N_+ - N_-}{N_+ + N_-} \quad \Delta f = f_{\text{Hg}} \left| \frac{\gamma_n}{\gamma_{\text{Hg}}} \right| - f_{\text{RF}}$$

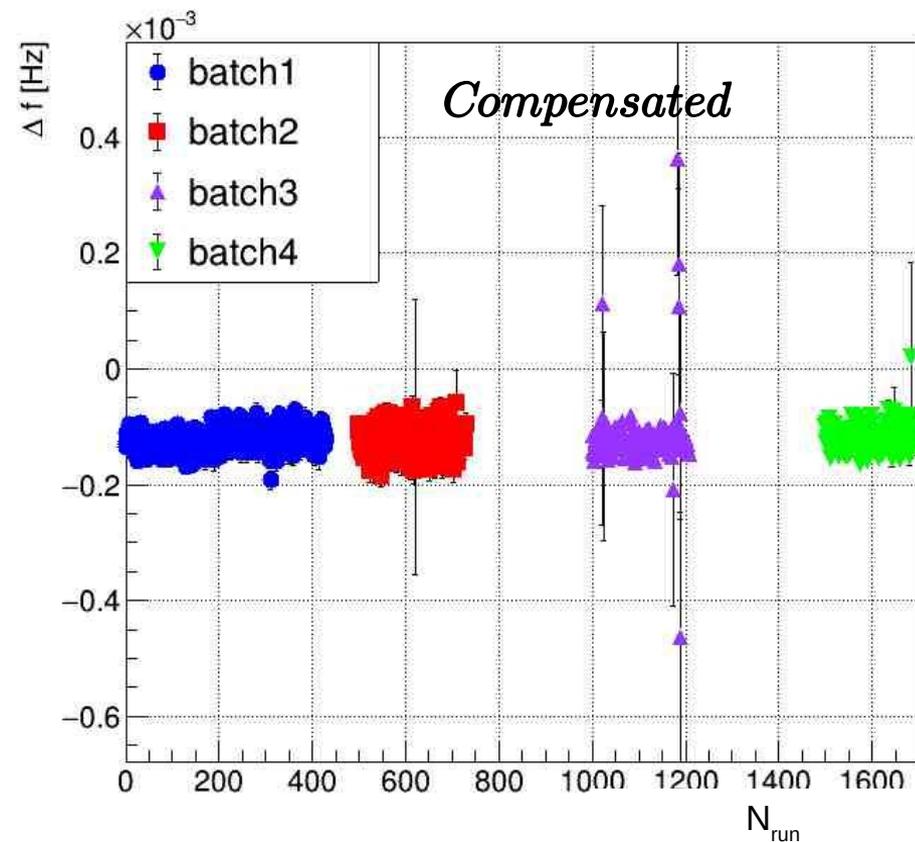
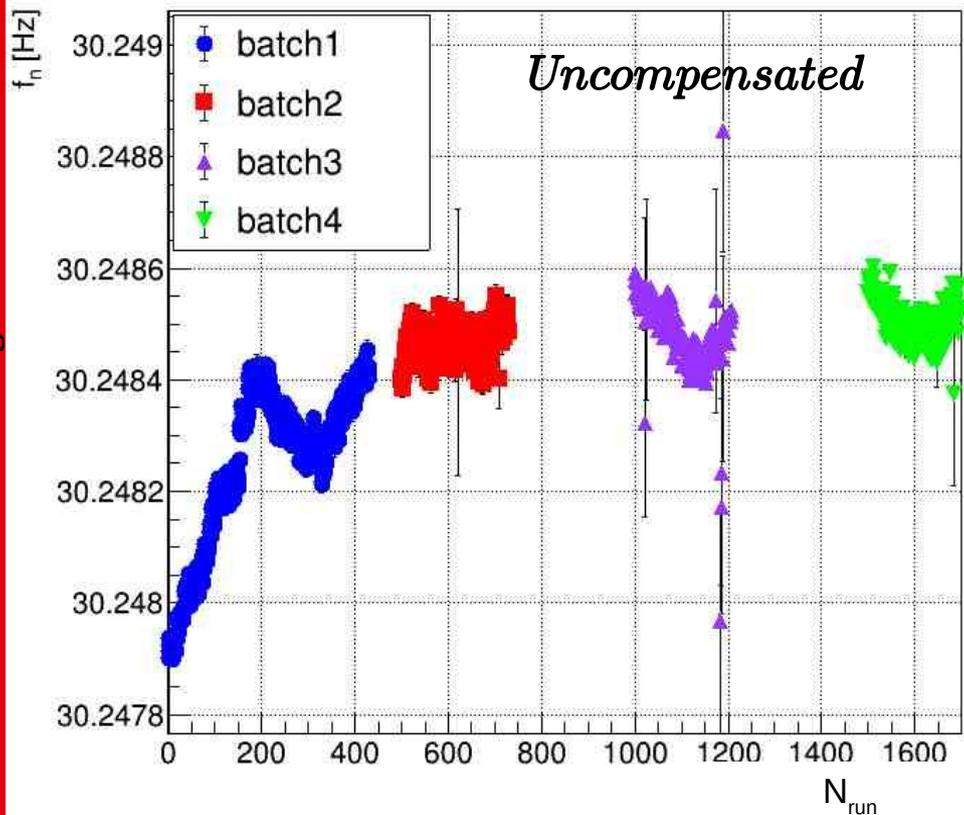
Fit formula:

$$\mathcal{A}(\Delta f) = A_a - |\alpha_a| \cos(2\pi T \Delta f - \phi_a)$$

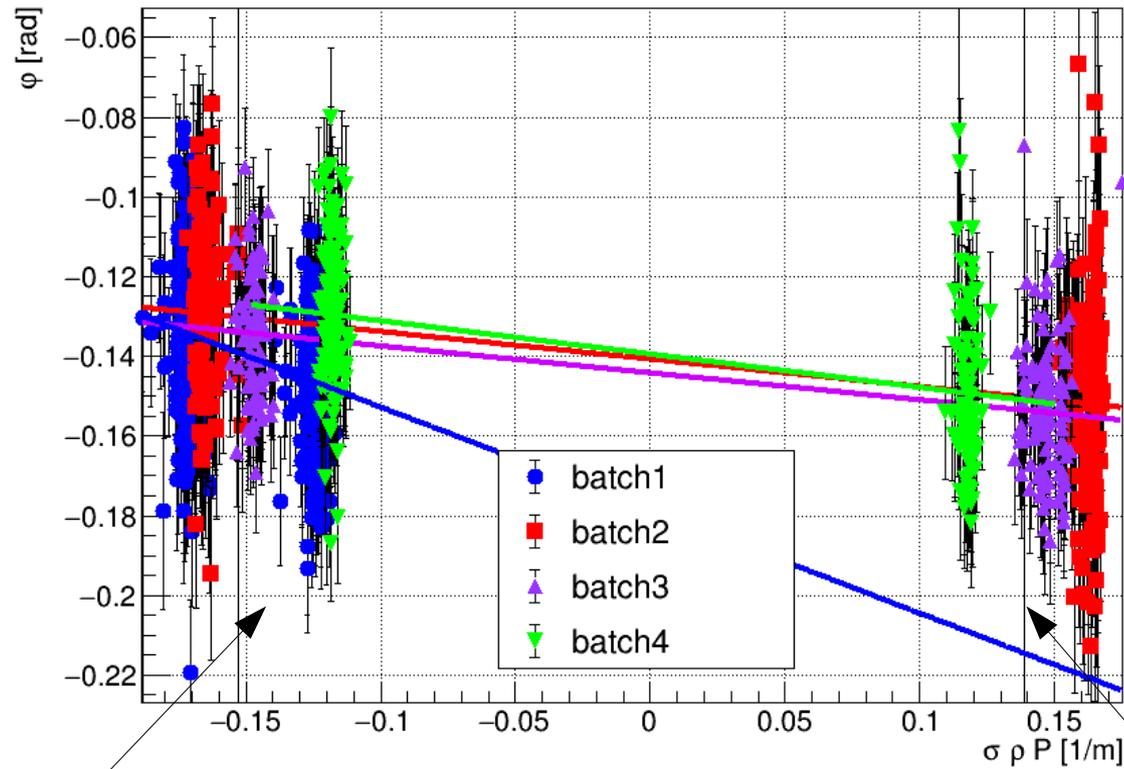
$$T = 182.5 \text{ s}$$

$$\varphi_n(\Delta f) = 2\pi T \Delta f - \text{sgn}(\Delta f) a \cos\left(\frac{A_a - \mathcal{A}(\Delta f)}{|\alpha_a|}\right)$$

# Mercury compensation



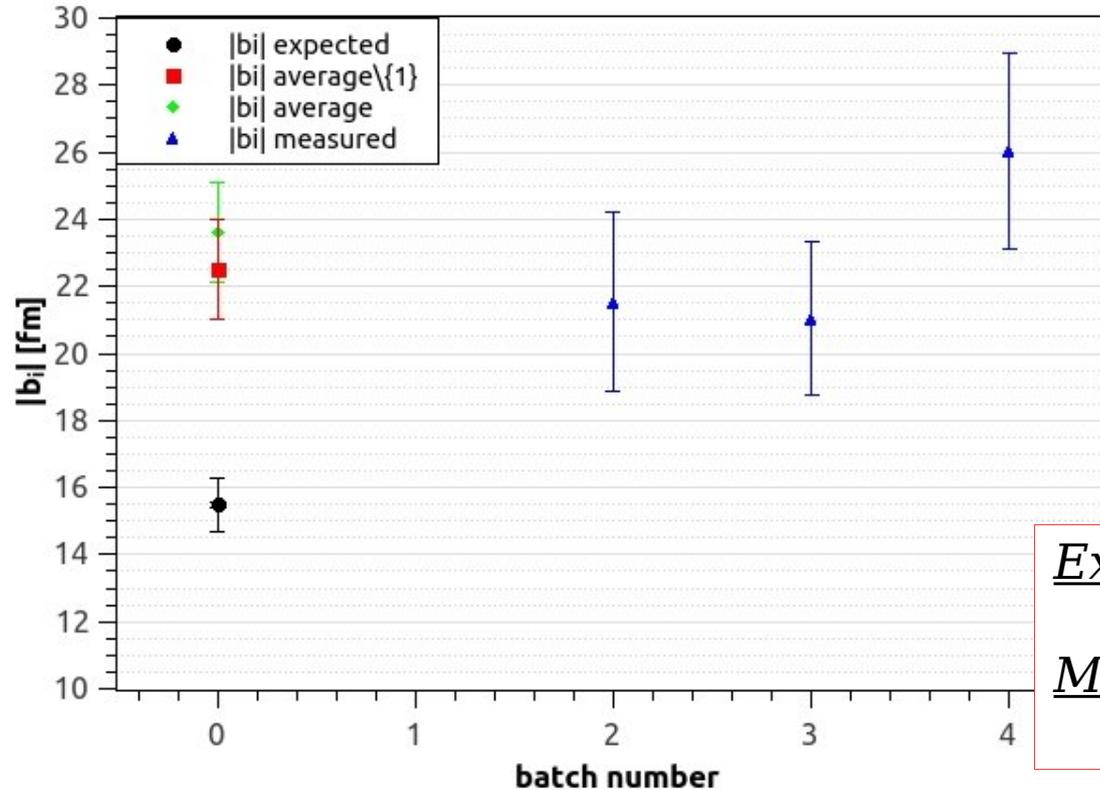
# Phase $\varphi_n$ vs $\sigma\rho P$



Batch	$\frac{d\varphi}{d\sigma\rho P}$ [rad]	error [rad]
1	-2.59E-1	3.48E-2
2	-6.91E-2	8.60E-3
3	-6.75E-2	7.32E-3
4	-8.35E-2	9.32E-3

$(T_1=100s)$

# Results for $b_i$



$$|b_i| = \left| \frac{\frac{d\varphi}{d\sigma\rho P} \times \frac{\sigma}{T}}{\gamma_n \cdot 2.49E-15 \text{ Tm}^2} \right|$$

$$\begin{aligned} \sigma &= 2.6E-17 \text{ m}^2 \\ T &= 182.5 \text{ s} \\ T_1 &= 100 \text{ s} \\ \gamma_n &= -1.83247E8 \end{aligned}$$

Expected value:

$$|b_i| = 15.5(8) \text{ fm}$$

Measured values:

$$|b_i| = 22.51 \text{ fm} \pm 1.49 \pm \dots$$

# XS problem

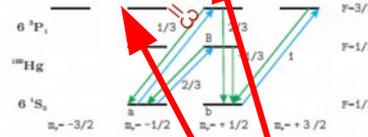


## Absorption cross section Hg-199



- Natural absorption cross section (Budker, Atomic Physics)
- Doppler broadening due to thermal movement of  $^{199}\text{Hg}$  → Doppler-width

$$\sigma_0 = \frac{\lambda^2}{2\pi} \frac{2J'+1}{2J+1} \frac{\gamma_p}{\gamma_{\text{tot}}} = 2/3 \gamma_{\text{tot}} = 2 \times 10^{-14} \text{m}^2$$



$$\Gamma_D = \frac{2\pi}{\lambda} \sqrt{\frac{2k_B T}{M}} \approx 3.0 \text{GHz}$$

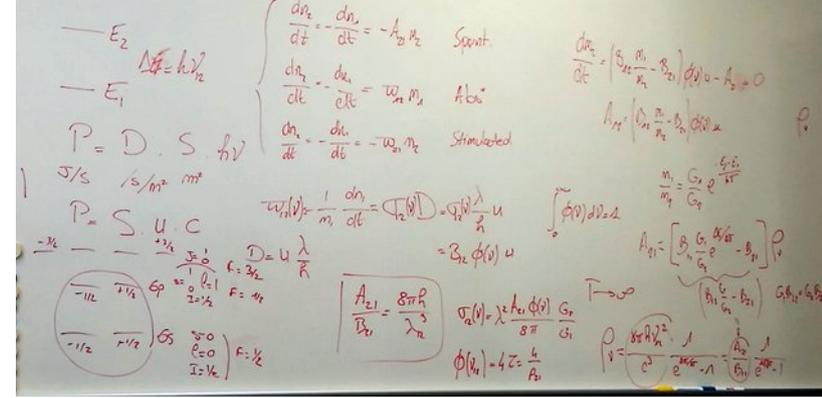
$$\sigma_D = \frac{\sigma_0 \sqrt{\pi}}{2} \frac{\gamma_p}{\Gamma_D}$$

unperturbed line width

$$\sigma_{199\text{Hg}} = 3.8 \times 10^{-13} \text{cm}^2$$

$\lambda_{\text{Hg199}} = 253.7 \text{nm}$   
 $\gamma_{\text{Hg199}} = 2\pi \cdot 1.3 \text{MHz}$   
 $T = 300 \text{K}$   
 $M = 199 \text{u} \approx 187 \text{GeV}/c^2$   
 $k_B = 8.617 \times 10^{-5} \text{eV/K}$

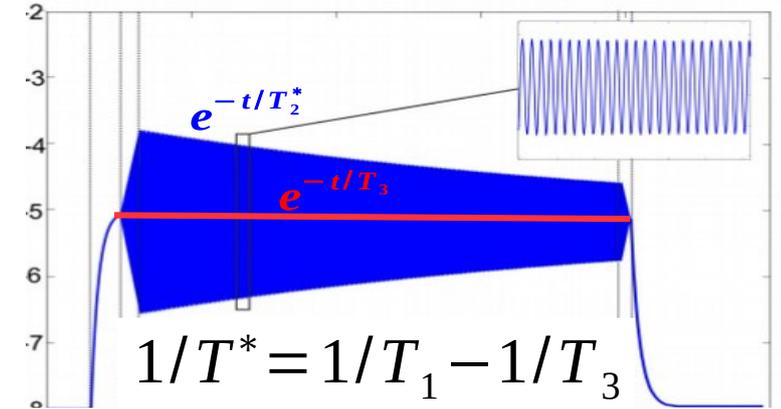
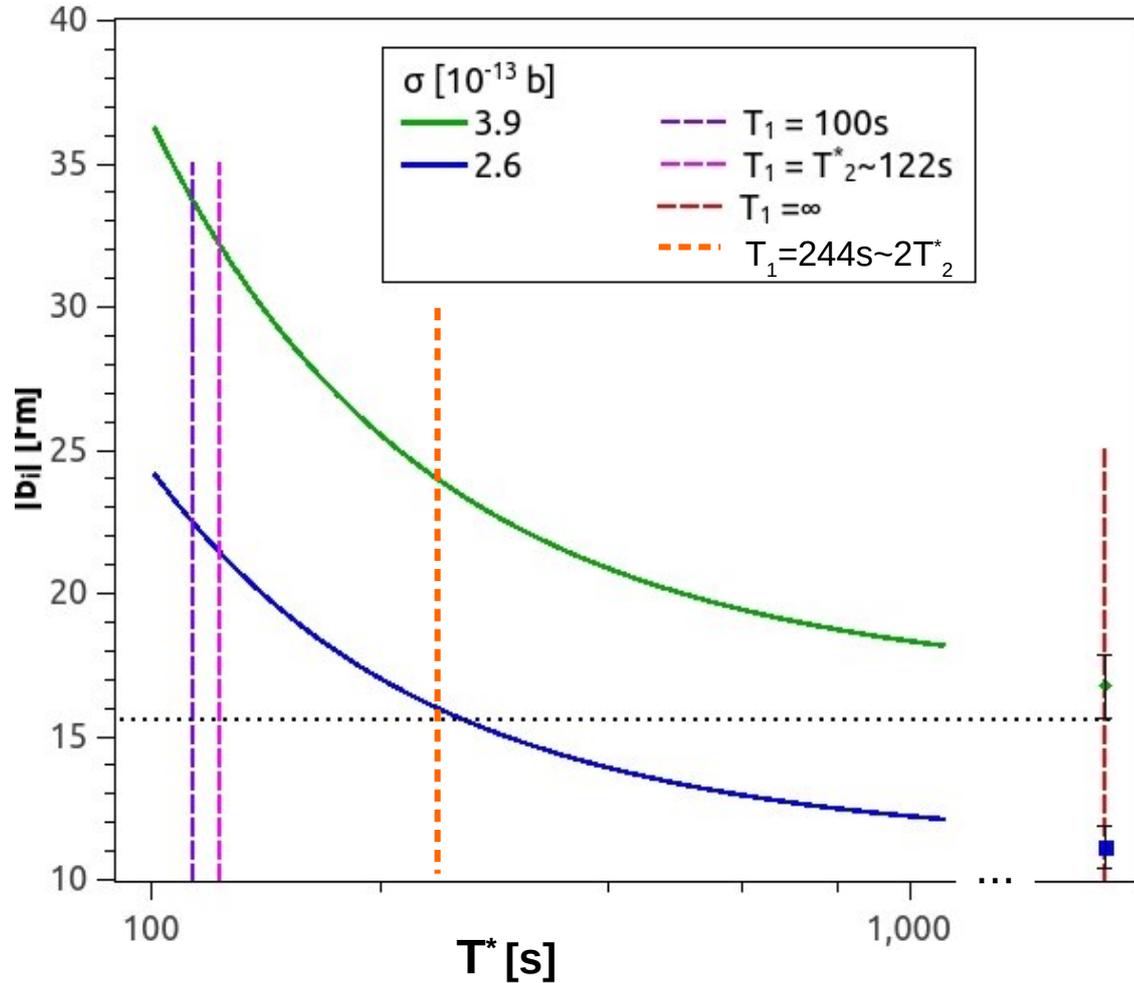
(Philipp's talk Bern)



Calculation of the mercury absorption XS (Arnaud's talk Bern)

**Multiplicative factor not agree upon => call for expert**

# Results for $b_i$ vs $T_1$

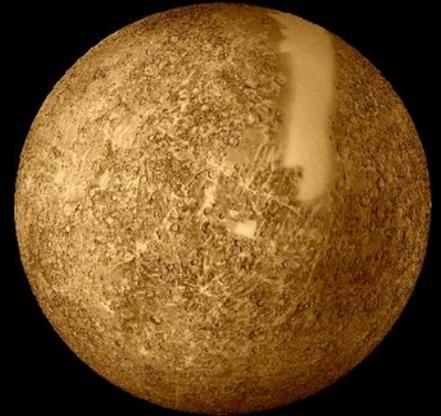


What is  $T_1$ ?  
Measured?

# Conclusion

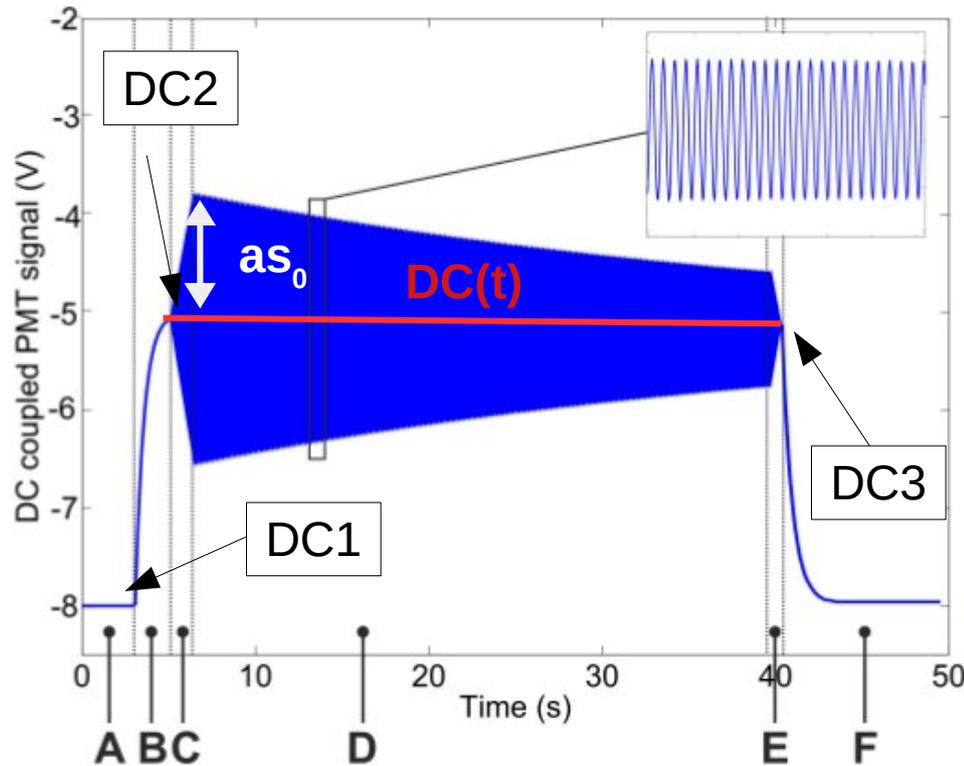
- ✓ systematic effect evaluated
- ✓ Non zero measurement
- ✓  $b_i$  right order of magnitude
  - ? values for  $T_1$
  - ? waiting for  $\sigma$
- ⊗ sign determination still on going

*questions*



# Back up slides

# Density Polarisation: $\rho P_{\text{eff}}$



Generic formula:

$$\sigma \rho P = \frac{1}{L} \text{ash}\left(\frac{as}{DC_2}\right) \quad L = 0.47 \text{ m}$$

Effective value:

$$(\sigma \rho P)_{\text{eff}} = \frac{1}{LT} \int \text{ash} \frac{As(t)}{DC(t)} dt$$

Assumptions:

$$\frac{As(t)}{DC(t)} \equiv \frac{as_0}{DC_2} \times e^{-t/T^*}$$

$$As(t) = as_0 \times e^{-t/T_1}$$

$$T_3 = T / \ln \left( \frac{DC_2 - DC_1}{DC_3 - DC_1} \right)$$

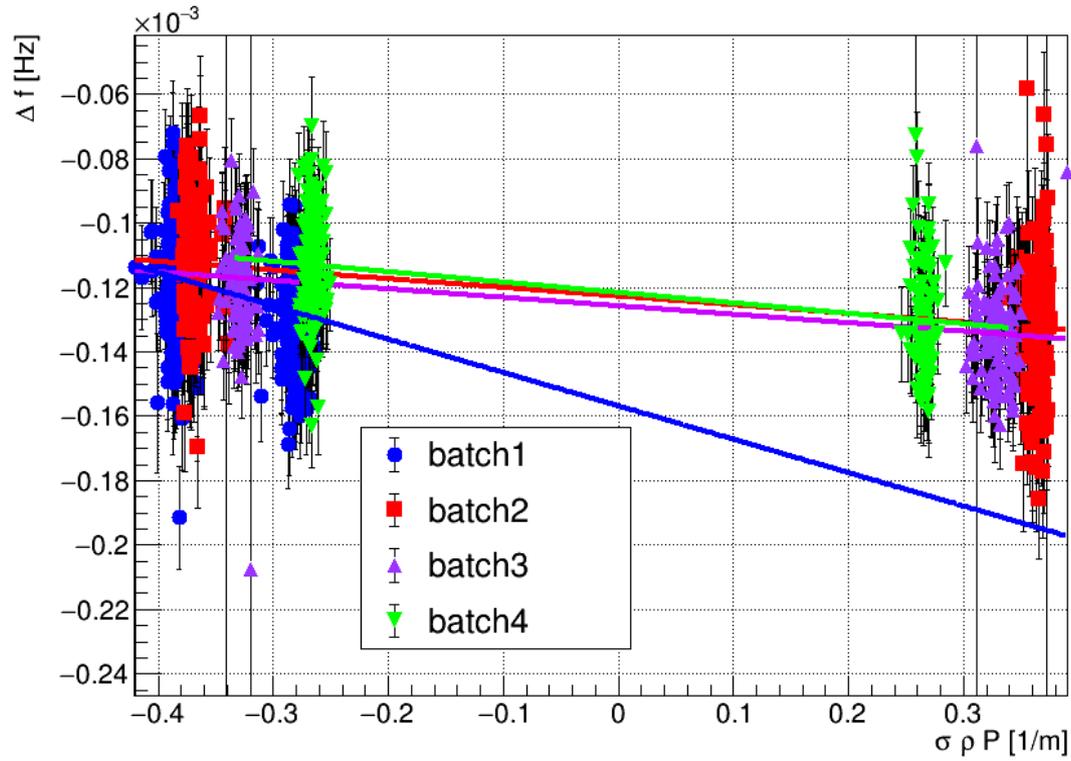
$$T = 182.5 \text{ s}$$

$$T_1 \sim 100 \text{ s}$$

$$T^{*-1} = 1/T_1 - 1/T_3$$

$$(\sigma \rho P)_{\text{eff}} \sim \frac{as_0 T^*}{DC_2 L T} (1 - e^{-T/T^*})$$

# Fit each batch individually



Batch	$\frac{d \Delta f}{d \sigma \rho P}$ [mHz]	error [mHz]
1	-2.25E-4	3.03E-5
2	-6.02E-5	7.50E-6
3	-5.88E-5	6.39E-6
4	-7.28E-5	8.13E-6

$(T_1=100s)$

# Data rejection

- $f_{\text{SourceWait}_s} > 40$
- $f_{\text{SourceWait}_s} < 1$
- batch1 1<sup>st</sup> data point  
(density too far from average value)

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< 68 data point rejected over more than 1150

# DC\_Offset=0 assumption

*“The term DC\_Offset accounts for light that cannot be absorbed by the Hg atoms in the enriched  $^{199}\text{Hg}$  sample. For example, this is light which is emitted by Hg isotopes in the  $^{204}\text{Hg}$  bulb different than the ones present in the enriched  $^{199}\text{Hg}$  sample. The laser light source tuned to the center of an absorption line closely approximates the ideal condition for which we define  $\text{DC\_Offset} = 0 \text{ V}$ . “*

# $\alpha$ and $\beta$ values

- *“ $\beta$ : Light absorption by isotopes other than  $^{199}\text{Hg}$  with modified cross section due to different or missing hyperfine splitting”*
- *“Here the small linewidth of the laser justifies the assumption of  $\alpha = 1$  and parameter  $\beta = 1.14$  was calculated using the Hg vapor isotopic composition as given in the datasheet of the supplier of the enriched Hg sample.”*