application brief AB20-3A

replaces AN1149-3A

Advanced Electrical Design Models

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Diode Equation Forward Voltage Model

Traditionally, the forward current versus forward voltage characteristics of a p-n junction diode have been expressed mathematically with the "Diode Equation" below.

$$I_F = I_O\left[\exp\left(\frac{qV_F}{nkT}\right) - 1\right] \qquad (3.1A)$$

Where:

V_F = forward voltage, V

I_F = forward current, A

n = ideality factor, $1 \leq n \leq 2$

I_O = reverse saturation current, A

T = temperature, °K

k = Boltzmann constant, 1.3805 x e-23 joule/°K

q = electron charge, 1.602 x e-19 coulomb

Note: at room temperature (25 °C), kT/q = 0.02569 V.

The reverse saturation current, I_O, varies by several orders of magnitude over the automotive temperature range so this effect must be included to properly model the forward characteristics of the LED lamp over temperature.

For forward voltage, V_F, greater than a few hundred millivolts, the exponential term predominates and the equation can be re-written as:

$$I_F \cong I_O \exp\left(\frac{qV_F}{nkT}\right)$$

$$V_F \cong \frac{nkT}{q} \ln\left(\frac{I_F}{I_O}\right)$$

The diode equation approximately models the low current (> 1 *µA*) performance of an LED emitter. However, at forward currents above a few mA, the ohmic losses must be included to accurately model the forward voltage. Thus, the diode equation becomes:

$$V_F \cong \frac{nkT}{q} \ln\left(\frac{I_F}{I_O}\right) + R'_S I_F \qquad (3.2A)$$

Where:

R'_S = internal series resistance, ohms

The values for the diode equation model can be calculated by using three test currents (I_{F1}, I_{F2}, and I_{F3}, such that $I_{F1} < I_{F2} < I_{F3}$). Then, the values of n, IO, and R´S would generate an equation that intercepts the forward characteristics of at these points: (I_{F1}, V_{F1}), (I_{F2}, V_{F2}), and (I_{F3}, V_{F3}) such as shown in Figure 3.1A. The equations for n, I_O, and R'_S are shown below:

$$n = \frac{I_{F3}(V_{F2} - V_{F1}) - I_{F2}(V_{F3} - V_{F1}) + I_{F1}(V_{F3} - V_{F2})}{\frac{kT}{q}\left[I_{F3}\ln\left(\frac{I_{F2}}{I_{F1}}\right) - I_{F2}\ln\left(\frac{I_{F3}}{I_{F1}}\right) + I_{F1}\ln\left(\frac{I_{F3}}{I_{F2}}\right)\right]} \qquad (3.3A)$$

$$R'_S = \frac{V_{F3}\ln\left(\frac{I_{F2}}{I_{F1}}\right) - V_{F2}\ln\left(\frac{I_{F3}}{I_{F1}}\right) + V_{F1}\ln\left(\frac{I_{F3}}{I_{F2}}\right)}{I_{F3}\ln\left(\frac{I_{F2}}{I_{F1}}\right) - I_{F2}\ln\left(\frac{I_{F3}}{I_{F1}}\right) + I_{F1}\ln\left(\frac{I_{F3}}{I_{F2}}\right)} \qquad (3.4A)$$

$$I_O = \frac{I_{F1}}{\exp\left[\frac{V_{F1} - R'_S I_{F1}}{\frac{kT}{q}n}\right]} \qquad (3.5A)$$

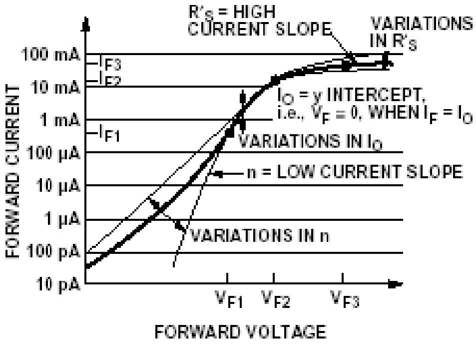

Figure 3.1A. Diode Equation Forward Voltage Model for LED Emitter (Semi-Log Scale).

Figure 3.2A shows how the diode equation model compares to the forward current versus forward voltage curve shown in AB20-3, Figure 3.8.

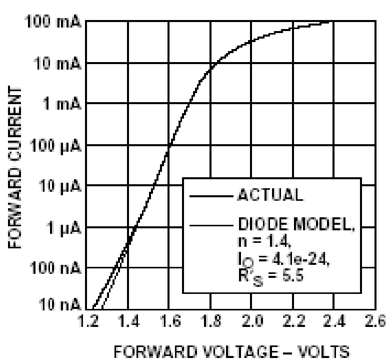

Figure 3.2A. Diode Equation Forward Voltage Model for HPWA-xHOO LED Emitter Shown in Figure 3.8 (Semi-Log Scale).

Using the values of the nominal forward voltage at the three test currents in Equations #3.3A, #3.4A, and #3.5A would generate the typical diode equation forward voltage model.

$$(I_{F1}, V_{F1\ nom}), (I_{F2}, V_{F2\ nom}), (I_{F3}, V_{F3\ nom}) \Rightarrow (n_{\ nom}, I_{O\ nom}, R'_{S\ nom})$$

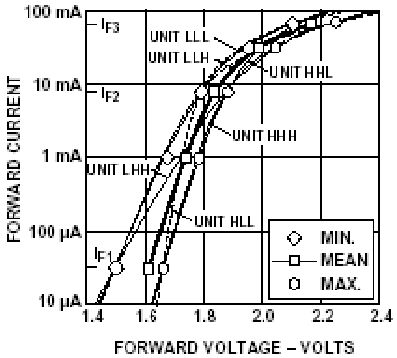

Figure 3.3A. Worst-Case Diode Equation Forward Voltage Models for LED Emitters. Note Graph Shows Forward Voltage Variations for LED Emitters from a Single Forward Voltage Category, Tested at I_F = *70 mA.*

Since there is little correlation between the forward voltages at each test condition, there are eight possible worst-case permutations of forward voltage at the three test currents. As shown in Figure 3.3A, these eight combinations of forward voltage can be used with Equations #3.3A, #3.4A, and #3.5A to generate eight different diode equation forward voltage models (n, I_O, and R'_S):

$$(I_{F1}, V_{F1\ min}), (I_{F2}, V_{F2\ min}), (I_{F3}, V_{F3\ min}) \Rightarrow (n_{\ LLL}, I_{O\ LLL}, R'_{S\ LLL})$$

$$(I_{F1}, V_{F1\ min}), (I_{F2}, V_{F2\ min}), (I_{F3}, V_{F3\ max}) \Rightarrow (n_{\ LLH}, I_{O\ LLH}, R'_{S\ LLH})$$

$$(I_{F1}, V_{F1\ min}), (I_{F2}, V_{F2\ max}), (I_{F3}, V_{F3\ min}) \Rightarrow (n_{\ LHL}, I_{O\ LHL}, R'_{S\ LHL})$$

$$(I_{F1}, V_{F1\ min}), (I_{F2}, V_{F2\ max}), (I_{F3}, V_{F3\ max}) \Rightarrow (n_{\ LHH}, I_{O\ LHH}, R'_{S\ LHH})$$

$$(I_{F1}, V_{F1\ max}), (I_{F2}, V_{F2\ min}), (I_{F3}, V_{F3\ min}) \Rightarrow (n_{\ HLL}, I_{O\ HLL}, R'_{S\ HLL})$$

$(I_{F1}, V_{F1\ max}), (I_{F2}, V_{F2}\ min), (I_{F3}, V_{F3\ max}) \Rightarrow$
$(n_{HLH}, I_{O\ HLH}, R'_{S\ HLH})$

$(I_{F1}, V_{F1\ max}), (I_{F2}, V_{F2\ max}), (I_{F3}, V_{F3}\ min) \Rightarrow$
$(n_{HHL}, I_{O\ HHL}, R'_{S\ HHL})$

$(I_{F1}, V_{F1\ max}), (I_{F2}, V_{F2\ max}), (I_{F3}, V_{F3\ max}) \Rightarrow$
$(n_{HHH}, I_{O\ HHH}, R'_{S\ HHH})$

In most situations, the worst-case range of forward current and forward voltage can be estimated with only two permutations of the diode equation model:

$$V_{F\ min} = \text{VDIODE}\ (I_F, n_{LLL}, I_{O\ LLL}, R'_{S\ LLL}) = \text{VDIODE}\ (I_F, n_{MIN}, I_{O\ MIN}, R'_{S\ MIN})$$

$$V_{F\ max} = \text{VDIODE}\ (I_F, n_{HHH}, I_{O\ HHH}, R'_{S\ HHH}) = \text{VDIODE}\ (I_F, n_{MAX}, I_{O\ MAX}, R'_{S\ MAX})$$

For analyzing the operation of an electronic circuit, it is convenient to be able to write the electrical forward characteristics of a component both in terms of forward voltage as a function of forward current as well as forward current as a function of forward voltage. The difficulty in using the diode equation (with the R'_S term) is that I_F as a function of V_F can only be solved through an iterative process. In addition, the reverse saturation current, I_O, varies by several orders of magnitude over the automotive temperature range so this effect must be included to properly model the forward characteristics of the LED emitter over temperature.

Advanced Thermal Modeling Equations

Note that, Equations #3.3 in AB20-3 or #3.6 in AB20-3 can be combined with Equation #3.9 in AB20-3 to derive the maximum DC forward current, $I_{F\ MAX}$, versus ambient temperature, T_A, and thermal resistance, $R\theta_{JA}$, shown in Figure 4 of the SuperFlux LED Data Sheet.

$$T_{J\ MAX} \cong T_A + R\ \theta_{JA}\ I_{F\ MAX}\ V_{F\ MAX} \cong T_A + R\ \theta_{JA}\ I_{F\ MAX}\ (V_{O\ HH} + R_{S\ HH} I_{F\ MAX})$$

Or written as a standard quadratic equation:

$$R\theta_{JA} R_{S\ HH} I_{F\ MAX}^2 + R\theta_{JA} V_{O\ HH} I_{F\ MAX} + T_A - T_{J\ MAX} \cong 0$$

Thus, the positive root solution of $I_{F\ MAX}$ is equal to:

$$I_{F\ MAX} \cong \frac{-V_{O\ HH} + \sqrt{V_{O\ HH}^2 - \frac{4R_{S\ HH}(T_A - T_{J\ MAX})}{R\theta_{JA}}}}{2R_{S\ HH}} \quad (3.6A)$$

Figure 3.4A shows Equation #3.6A graphed as a function of T_A and $R\theta_{JA}$ for an HPWA-xH00 LED emitter with a maximum expected forward voltage (i.e. $V_F = 2.67\ V$ *at 70 mA*). Values of $T_{J\ MAX} = 125\ °C$, $V_{O\ HH} = 1.83\ V$, and $R_{S\ HH} = 12$ *ohms* were used for Figure 3.4A. Note that Figure 3.4A is the same as Figure 4a, “HPWA-XX00 Maximum DC Forward Current vs. Ambient Temperature” graph, in the SuperFlux LED Data Sheet.

Equations #3.7 in AB20-3, #3.8 in AB20-3, and #3.9 in AB20-3 can be combined together in different ways to model the luminous flux (or luminous intensity) of LED emitters due to the effects of internal self-heating (i.e. $R\theta_{JA}P_D$) and ambient temperature. Equation #3.7A models the expected reduction in luminous flux due to internal self-heating compared to the instantaneous luminous flux (i.e. at initial turn-on) when the LED emitter is driven at a constant forward current at a constant ambient temperature. Equation #3.8A models the thermally stabilized luminous flux at any forward current compared to the instantaneous luminous flux prior to heating at a specified forward current and a constant ambient temperature. Equation #3.9A models the thermally stabilized luminous flux at any forward current compared to the thermally stabilized luminous flux at test conditions of $I_{F\ TEST}$, $V_{F\ TEST}$, and $R\theta_{JA\ TEST}$ at a constant ambient temperature. A good example of an application for Equation #3.9A is the normalized luminous flux versus forward current graph shown in Figure 3 of the SuperFlux LED Data Sheet. Finally, Equation #3.10A models the thermally stabilized luminous flux over temperature compared to the thermally stabilized luminous flux at test conditions of $I_{F\ TEST}$, $V_{F\ TEST}$, and $R\theta_{JA\ TEST}$, at 25°C. Note for Equations #3.8A, #3.9A, and #3.10A, that for forward currents over 30 mA, $m \approx 1.0$.

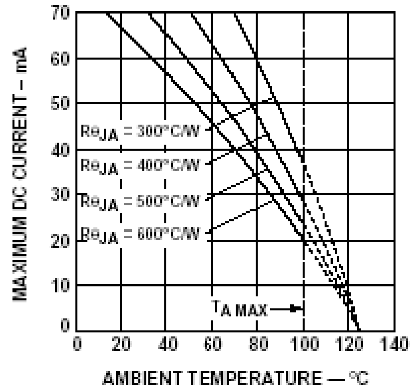

Figure 3.4A. Maximum DC Forward Current versus Ambient Temperature for HPWA-xxOO LED Emitter with Different System Thermal Resistances.

$$\begin{aligned}\Phi_V(R\theta_{JA},P_D) &\equiv \Phi_V(T_J = 25°C)\exp[k(R\theta_{JA}P_D)] \qquad (3.7A)\\ &\equiv \Phi_V(T_J = 25°C)\exp[k(R\theta_{JA}I_F V_F)]\end{aligned}$$

$$\Phi_V(I_F,V_F,\ R\theta_{JA}) \equiv \Phi_V(I_{F\,TEST},T_J = 25°C)\left[\frac{I_F}{I_{F\,TEST}}\right]^m \exp[k(R\theta_{JA}I_F V_F)] \qquad (3.8A)$$

$$\Phi_V(I_F,V_F,\ R\theta_{JA}) \equiv \Phi_V(I_{F\,TEST},V_{F\,TEST},\ R\theta_{JA\,TEST})\left[\frac{I_F}{I_{F\,TEST}}\right]^m \exp[k(R\theta_{JA}I_F V_F - R\theta_{JA\,TEST}I_{F\,TEST}V_{F\,TEST})] \qquad (3.9A)$$

$$\Phi_V(I_F,V_F,\ R\theta_{JA},\ T_A) \equiv \Phi_V(I_{F\,TEST},V_{F\,TEST},\ R\theta_{JA\,TEST},\ 25°C)\left[\frac{I_F}{I_{F\,TEST}}\right]^m \exp\{k[(T_A + R\theta_{JA}I_F V_F - R\theta_{JA\,TEST}I_{F\,TEST}V_{F\,TEST} - 25°C]\} \qquad (3.10A)$$

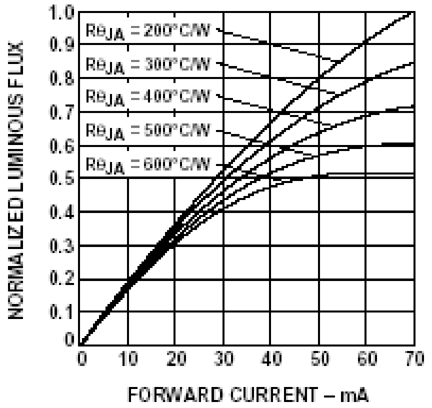

Figure 3.5A. Thermally Stabilized Luminous Flux versus DC Forward Current for HPWx-xHOO LED Emitter with Different System Thermal Resistances.

Figure 3.5A shows Equation #3.9A graphed as a function of I_F and $R\theta_{JA}$ for an HPWA-xH00 LED emitter with a nominal forward voltage (i.e., $V_F = 2.25\ V$ at *70 mA*). Values of $R\theta_{JA\ TEST} = 200$ *°C/W*, *m = 1.0*, *k = –0.0106*, $V_{O\ NOM} = 1.802\ V$, *and* $R_{S\ NOM} = 6.4$ *ohms* were used for Figure 3.5A. Note that Figure 3.5A is the same as Figure 3, “HPWA/HPWT-xx00 Relative Luminous Flux vs. Forward Current” graph, in the SuperFlux LED Data Sheet.

This section discussed the key concepts of modeling the electrical, optical, and thermal performance of LED signal lights. Equation #3.6A is a combination of Equations #3.3 in AB20-3 and #3.8 in AB20-3 that can be used to calculate the maximum forward current as a function of ambient temperature and thermal resistance. Note that this equation models Figure 4 (Maximum DC Forward Current versus Ambient Temperature) on the SuperFlux LED Data Sheet. Equations #3.7A, #3.8A, #3.9A, and #3.10A show different combinations of equations #3.7 in AB20-3, #3.8 in AB20-3, and #3.9 in AB20-3 in order to model various thermal effects on the light output of the emitter. Note that Equation #3.10A models Figure 3 (Normalized Luminous Flux versus Forward Current) on the SuperFlux LED Data Sheet.

Company Information

Lumileds is a world-class supplier of Light Emitting Diodes (LEDs) producing billions of LEDs annually. Lumileds is a fully integrated supplier, producing core LED material in all three base colors (Red, Green, Blue) and White. Lumileds has R&D development centers in San Jose, California and Best, The Netherlands. Production capabilities in San Jose, California and Malaysia.

Lumileds is pioneering the high-flux LED technology and bridging the gap between solid state LED technology and the lighting world. Lumileds is absolutely dedicated to bringing the best and brightest LED technology to enable new applications and markets in the Lighting world.

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